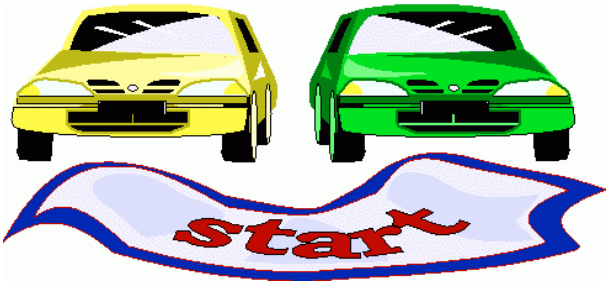
Sum Permutation



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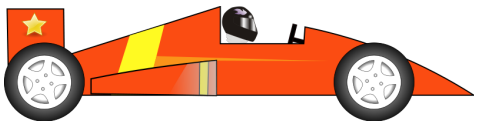
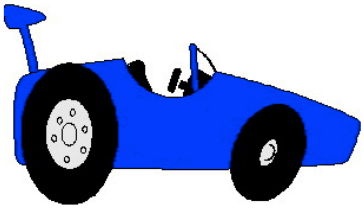
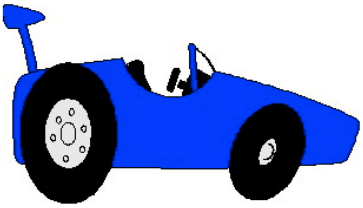
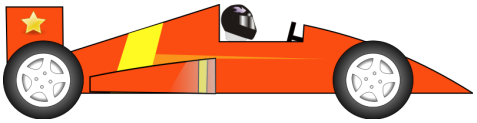








Sum Permutation

Step 1: Write down a 3 digit number any 3 DIFFERENT DIGITS from 1 to 9, (no repeats)

Step 1. ____ ____ ____

Step 2. Write down six different 2 digit numbers using the digits in the 3 digit number above with no digits repeated.

Example: If your number was 527 then 52 and 25 would be 2 of the six numbers.
55 would not be one of the 6 numbers.

____ ____

____ ____

____ ____

____ ____

____ ____

\+ ____ ____

Step 3. Find the total of the 6 numbers Total =

Step 4. Find the total of the 3 digits used in step 1. Add ____ + ____ + ____ = _____
Total

Step 5. Divide the total in step 3 by the total in step 4.

Your answer is _____

Try It Again
Sum Permutation

Step 1: Write down a 3 digit number any 3 DIFFERENT DIGITS from 1 to 9, (no repeats)

Step 1. ____ ____ ____

Step 2. Write down six different 2 digit numbers using the digits in the 3 digit number above with no digits repeated.

Example: If your number was 527 then 52 and 25 would be 2 of the six numbers.
55 would not be one of the 6 numbers.

____ ____

____ ____

____ ____

____ ____

____ ____

\+ ____ ____

Step 3. Find the total of the 6 numbers Total =

Step 4. Find the total of the 3 digits used in step 1. Add ____ + ____ + ____ = _____
Total

Step 5. Divide the total in step 3 by the total in step 4.

Your answer is _____

Sum Permutation

Step 1: Write down a 3 digit number any 3 DIFFERENT DIGITS from 1 to 9, (no repeats)

Step 2. Write down six different 2 digit numbers using the digits in the 3 digit number above with no digits repeated. If your number was 527 then 52 and 25 would be 2 of the six numbers. 55 would not be one of the 6 numbers.

Step 3. Find the total of the 6 two digit numbers.

Step 4. Divide this total by the sum of the 3 digits in the original 3 digit number.

The answer is 22.

Example 1

Step 1. 3 8 2

Step 2. 3 8
3 2
8 2
8 3
2 3
2 8

Step 3 The total is 286

Step 4 286 / (3 + 8 + 2) = 22

Example 2

Step 1. 1 9 6

Step 3. 1 9
1 6
9 1
9 6
6 1
6 9

Step 3 The total is 352

Step 4 352 / (1 + 9 + 6) = 22

Proof:

1. A three digit number with no digits repeated A B C

2. Six different 2 digit numbers from A B C with no repeated digits

A B	This two digit number can be expressed as 10A + B
A C	This two digit number can be expressed as 10A + C
B A	This two digit number can be expressed as 10B + A
B C	This two digit number can be expressed as 10B + C
C A	This two digit number can be expressed as 10C + A
C B	This two digit number can be expressed as 10C + B

3. 22A + 22B + 22C which equals 22 (A + B + C)

4. 22 (A + B + C) / (A + B + C)

5. The result is always 22.

Questions: Which if any of the following can be allowed and still have the effect work?

1. Can you use the digits 0 to 9 and still have the effect work.

2. Can you use repeats of the digits 1 to 9 (or 0 to 9) to make your 3 digit number. i.e. 119 or 223

3. Can you use repeats of the digits to make your two digit numbers. i.e. AA or BB

Make up a similar effect:

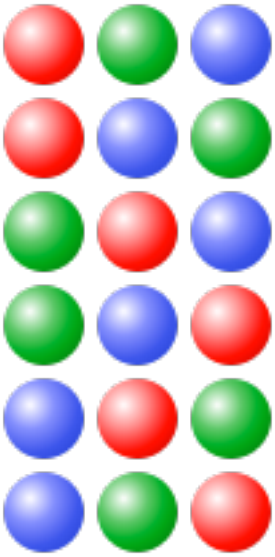
What changes to the procedures would you need to make if the original trick started with a 3 digit with no digits repeated using the digits 1 to 9. number and created 3 digits numbers with no digits repeated using the digits 1 to 9.

Background Information.

When I work with students I start with the instructional pages that introduce the concept of a Permutation. I project the pages on a screen to the class and talk them through the lecture pages. The definition of permutation presented here is based on **taking R objects** from a **set of N DIFFERENT objects** one at a time. We call this N choose R. The order the objects are taken in makes each permutation a different permutation. Permutations form the basis for the solutions to many problems in probability at the high school and college level.

A permutation of a set of 3 objects
where we take 2 of the objects
WE SAY THAT THiS IS 3 CHOSE 2

A permutation of a set of 3 objects
where we take all 3objects
WE SAY THAT THIS IS 3 CHOSE 3



The teacher lecture pages start by defining a permutation using a collection of race cars where each car is a different color. Examples are provided. I then develop a concept using the same race cars but with different numbers on the cars. Race cars work well as it is clear to student that a race has first. second and third place cars and the order they finish in matters. The numbers on the cars allow me to make multiple worksheets by just changing the numbers on the cars. It also leads towards the magic effect I want to use to end the lesson. I provide 2 student activity pages to allow the student to develop the concept.of 3 choose 2 with the numbered cars. You could finish the instruction and present the magic effect at this point and conclude the lesson if you wanted to.

I am interested in having student extend this concept to similar problems. To help get students on that path I have the students find the permutations of 3 objects chosen from a set of 3 objects. We call this 3 choose 3. I help the students further develop the skill of systematic counting to help the student in finding all the permutations. As an extension I may also help them find the permutations of 4 objects chosen from a set of 4 objects, We call this 4 choose 4. This level of systematic counting may be a challenge but the activity page I use reduces the chance of failure.

At this point I present the magic effect with the 3 choose 2 permutation of any three digit number they select using t3 different digits from 1 to 9. I have them work in groups of 2 or 3 students. Each group of students decides on the same number and does their work independently but checks their work with the group. THIS CUTS DOWN ON THE "It did not work for me" issue you will have with some students if you let each student work alone.

I have them do the effect a second time using a different 3 digit number and I provide a second page for them to use.

If your students have algebraic skills then I would introduce the proof that the answer will always be 22.

I would then give them the followup page with 3 questions and an extension.

Questions: Which if any of the following can be allowed and still have the effect work?

1. Can you use the digits 0 to 9 and still have the effect work.
2. Can you use repeats of the digits 1 to 9 (or 0 to 9) to make your 3 digit number. i.e. 119 or 223
3. Can you use repeats of the digits to make your two digit numbers. i.e. AA or BB

Make up a similar effect:

What changes to the procedures would you need to make if the original trick started with a 3 digit with no digits repeated using the digits 1 to 9. number and created 3 digits numbers with no digits

For some of your better students they may be able to extend the 3 choose 2 effect to a 3 choose 3 effect. In the 3 chose 3 effect the answer is always 222.

Permutations

A permutation is a collection of objects where each object was taken form an original collection of several different objects one at a time. Each object taken cannot be taken a second time and the order the objects are taken in makes that permutation different then the same collects taken in a different order.

A permutation is **a collection of different objects** in a specific order.
The **order** of the arrangement **is important** !

I have a collection of 3 cars, each with a **different color.** One blue, one green and one red.

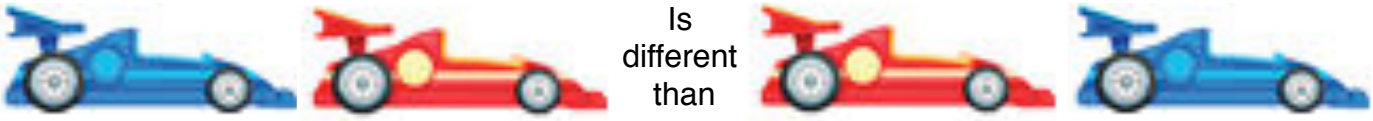
I want to make a new collection by picking only **2 of the cars**. I will select a first car from the original collection and then take a second car from the 2 cars that are left in the collection.

T0 show the collection you could show the two objects selected in the order they were selected in.

or write the colors in english Blue, Green

or use abbreviations B G

If the cars are selected in a different order that will be a different collection.



Is different than

Blue Red is different than Red Blue

I have a collection of 3 cars, each with a **different color.** One blue, one green and one red.

I want to make a new collection by picking only **2 of the cars**. I will select a first car from the original collection and then take a second car from the 2 cars that are left in the collection.

How many different collections can I make?

A **blue** car could be picked **first**
and then the **green** car second

First Second

A **blue** car could picked **first**
and then a **red** car **second**

First Second

A **green** car could come in **first**
and then the **blue** car **second**

First Second

A **green** car could come in **first**
and then a **red** car **second**

First Second

A **red** car could come in **first**
and then the **blue** car **second**

First Second

A **red** car could come in **first**
and then a **green** car **second**

First Second

These are the only permutations than can be made with 2 of the 3 cards in the collection

Blue Green	Blue Red
Green Blue	Green Red
Red Blue	Red Green

Permutations

The permutations of 3 objects when 2 objects are chosen 1 at a time.

There are **6 different permutations** of a collection of **3 different colored cars** when **you take 2 different colored cars from the original collection.** The word permutation tells us that the order you take the balls in makes a difference. Red Blue is a different permutation than Red Blue

I selected a **blue car to finish first** and then decided if a green or red car would be taken second.
I selected a **green car to finish first** and then decided if a blue or red car would be taken second.
I selected a **red car to finish first** and then decided if a blue or green car would be taken second.

First	Second
	
	
	
	
	
	

First	Second
B	G
B	R
G	B
G	R
R	B
R	G

The 6 different permutations of 3 different colored cars when you take 2 different colored cars one at a time are shown to the left.

The order they are taken in makes a difference

The permutations of 3 objects when 2 objects are chosen 1 at a time.

The race cars shown below are going to a race. Each of the cars has a different number on the car. The first place winner will get $100 and the second place car will get $ 50.

How I make the table of permutations.

I selected the **blue car to finish first** and then decided who finished second.
I selected the **green car to finish first** and then decided who finished second.
I selected the **red car to finish first** and then decided who finished second.

First	Second
	
	
	
	
	
	

First	Second
7	4
7	3
4	7
4	3
3	7
3	4

The 6 different ways the numbered cars could finish first and second

The 6 different permutations of 3 different numbers when you take 2 different numbers one at a time are listed to the left.

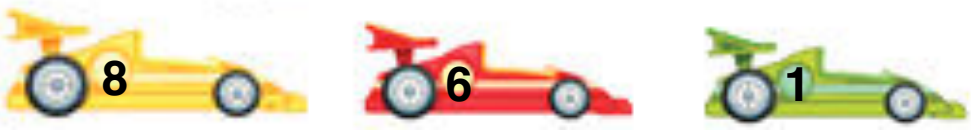
The order they are finish in makes a difference

There are **6 different permutations** of a collection of **3 different numbered cars** when **you take 2 different numbered cars one at a time from the original collection.** The word permutation tells us that the order the cards finish in makes a difference. A 7 4 finish is different permutation than a 4 7 finish.

Problem 1

The permutations of 3 objects when 2 objects are chosen 1 at a time.

The race cars shown below are going to a race. Each of the cars has a different number on the car. The first place winner will get $100 and the second place car will get $ 50.



How to make the table of permutations.

Select a **car to finish first** and then decide who finishes second.
Select different **car to finish first** and then decide who finishes second.
Select a different **car to finish first** and then decide who finishes second. .

First	Second

Problem 2

The permutations of 3 objects when 2 objects are chosen 1 at a time.

The race cars shown below are going to a race. Each of the cars has a different number on the car. The first place winner will get $100 and the second place car will get $ 50.

How to make the table of permutations.

Select a **car to finish first** and then decide who finishes second.
Select different **car to finish first** and then decide who finishes second.
Select a different **car to finish first** and then decide who finishes second. .

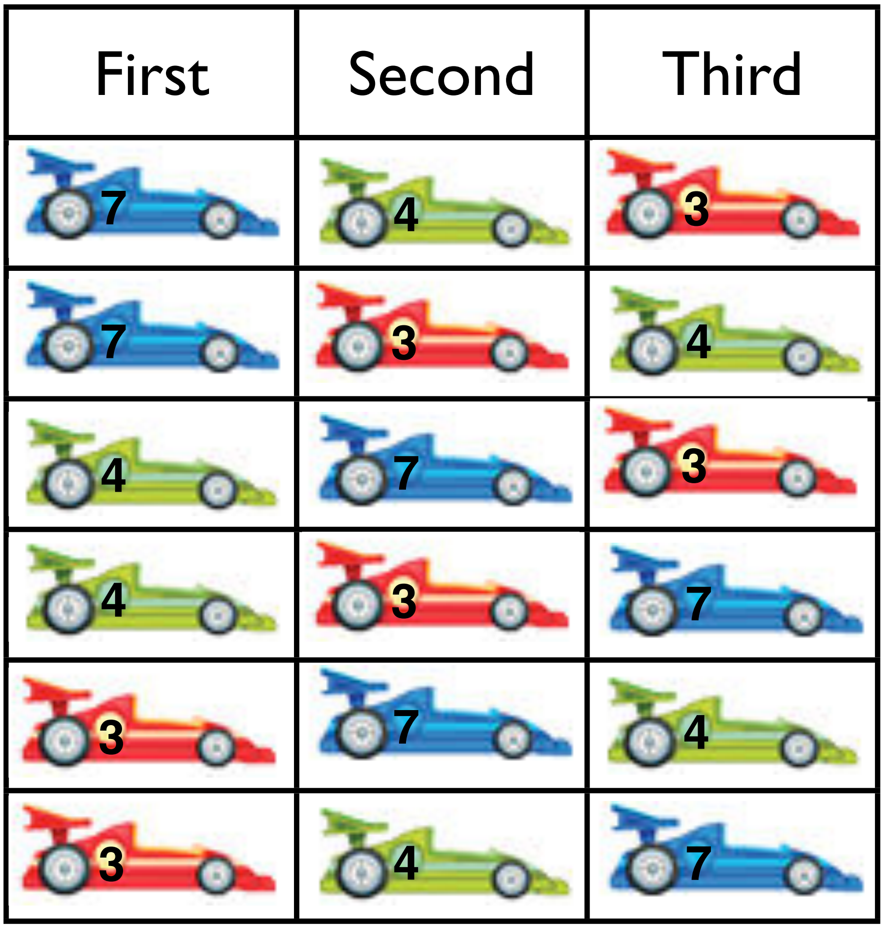
First	Second

The permutations of 3 objects when 3 objects are chosen 1 at a time.

The race cars shown below are going to a race. Each of the cars has a different number on the car. The **first place** winner will get $100 , the **second place car will get $ 50 and the third place car will get $ 10**.

How I make the table of permutations.

I selected the **blue car to finish first** and then decided who finished second and third.
I selected the **green car to finish first** and then decided who finished second and third.
I selected the **red car to finish first** and then decided who finished second and third.



First	Second	Third
7	4	3
7	3	4
4	7	3
4	3	7
3	7	4
3	4	7

There are **6 different permutations** of a collection of **3 different numbered cars** when **you take 3 different numbered cars one at a time from the original collection.** The word permutation tells us that the order the cards finish in makes a difference. A 7 4 finish is different permutation than a 4 7 finish.

Problem 3

The permutations of 3 objects when 3 objects are chosen 1 at a time.

The race cars shown below are going to a race. Each of the cars has a different number on the car. The first place winner will get $100 and the second place car will get $ 50.

How to make the table of permutations.

Select a **car to finish first** and then decide who finishes second.
Select different **car to finish first** and then decide who finishes second.
Select a different **car to finish first** and then decide who finishes second. .

First	Second	Third

Problem 4

The permutations of 3 objects when 2 objects are chosen 1 at a time.

The race cars shown below are going to a race. Each of the cars has a different number on the car. The first place winner will get $100 and the second place car will get $ 50.

How to make the table of permutations.

Select a **car to finish first** and then decide who finishes second.
Select different **car to finish first** and then decide who finishes second.
Select a different **car to finish first** and then decide who finishes second. .

First	Second	Third

The permutations of 4 objects when 4 objects are chosen 1 at a time.

4 race cars are numbered 1 , 2 , 3, and 4. They are going to race each other. The **first place winner will get $1000** , the **second place car will get $ 500 , the third place car will get $ 200 and the forth place car will get $ 50**.

List all the possible ways the prizes could be handed out. This means find all the possible permutations of the 4 numbers taken one at a time from the collection of 4 numbers

Using the pictures of the cars takes too much room. It is easier to the numbers 1 , 2 , 3, and 4.

How to make the table of permutations.

Select 1 and 2 and then decided who finished third and fourth.

1 2 ____ _____ 1 2 ____ _____

Select 1 and 3 and then decide who finishes third and fourth.

1 3 ____ _____ 1 3 ____ _____

Select 1 and 4 and then decide who finishes third and fourth.

1 4 ____ _____ 1 4 ____ _____

Select 2 and 1 and then decide who finishes third and fourth.

2 1 ____ _____ 2 1 ____ _____

Select 2 and 3 and then decide who finishes third and fourth.

2 3 ____ _____ 2 3 ____ _____

Select 2 and 4 and then decide who finishes third and fourth.

2 4 ____ _____ 2 4 ____ _____

Select 3 and 1 and then decide who finishes third and fourth.

3 1 ____ _____ 3 1 ____ _____

Select 3 and 2 and then decide who finishes third and fourth.

3 2 ____ _____ 3 2 ____ _____

Select 3 and 4 and then decide who finishes third and fourth.

3 4 ____ _____ 3 4 ____ _____

Select 4 and 1 and then decide who finishes third and fourth.

4 1 ____ _____ 4 1 ____ _____

Select 4 and 2 and then decide who finishes third and fourth.

4 2 ____ _____ 4 2 ____ _____

Select 4 and 3 and then decide who finishes third and fourth.

4 3 ____ _____ 4 3 ____ _____

Problem 5

The permutations of 4 objects when 4 objects are chosen 1 at a time.

4 race cars are numbered 1 , 2 , 3, and 4. They are going to race each other. The **first place winner will get $1000** , the **second place car will get $ 500 , the third place car will get $ 200 and the forth place car will get $ 50**.

List all the possible ways the prizes could be handed out. This means find all the possible permutations of the 4 numbers taken one at a tome from the collection of 4 numbers

Using the pictures of the cars takes too much room. It is easier to the numbers 1 , 2 , 3, and 4.

How to make the table of permutations.

1	2	____	_____	1	2	____	_____
1	3	____	_____	1	3	____	_____
1	4	____	_____	1	4	____	_____
2	1	____	_____	2	1	____	_____
2	3	____	_____	2	3	____	_____
2	4	____	_____	2	4	____	_____
3	1	____	_____	3	1	____	_____
3	2	____	_____	3	2	____	_____
3	4	____	_____	3	4	____	_____
4	1	____	_____	4	1	____	_____
4	2	____	_____	4	2	____	_____
4	3	____	_____	4	3	____	_____

Solution to problem 5

A **Permutation** of 1,2,3,4 taken 1 at a time

1,2,3,4	2,1,3,4	3,2,1,4	4,2,3,1
1,2,4,3	2,1,4,3	3,2,4,1	4,2,1,3
1,3,2,4	2,3,1,4	3,1,2,4	4,3,2,1
1,3,4,2	2,3,4,1	3,1,4,2	4,3,1,2
1,4,2,3	2,4,1,3	3,4,2,1	4,1,2,3
1,4,3,2	2,4,3,1	3,4,1,2	4,1,3,2