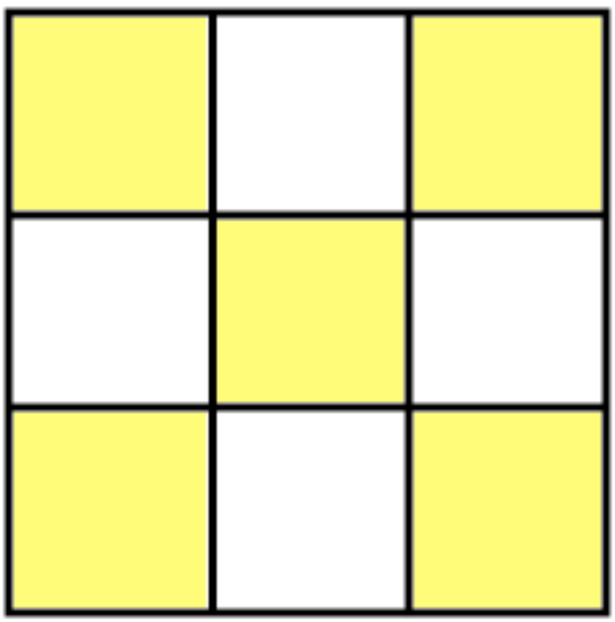
Square Deal

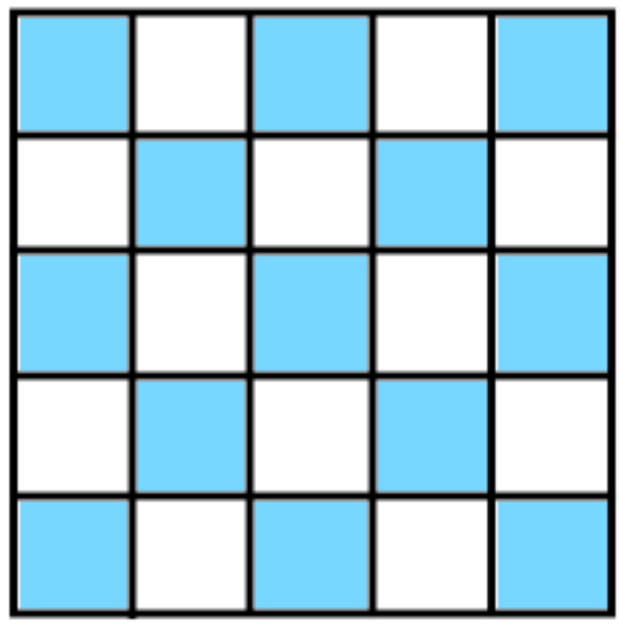
David and John were each given a square of equal areas. David divided his square into several smaller **yellow** squares each of which were of equal size. John divided his square into several smaller **blue** squares each of which were of equal size.

David and John wonder if the total area of the yellow squares and the area of the blue squares are equal.   Did they each get a square deal or does one of the boys shaded squares contain more area than the other?

David's Square



John's Square



Solution:

Let the area of both large squares be 1 square inch (sq. in.)

David's square with the smaller yellow squares has 9 small squares in it so the area of each yellow square is 1/ 9 th of the total area. There are 5 of these yellow squares so the total area of the yellow squares is $\frac{5}{9}$ sq. in

John's square with the smaller blue squares has 25 squares in it so the area of each blue square is 1/ 25 th of the total area. There are 13 of these blue squares so the total area of the yellow squares is 13/ 25 th of a sq. in

David's 5 yellow squares have a total area of $\frac{5}{9}$ sq. in

John's 13 blue squares have an area of $\frac{13}{25}$ sq. in

Which is larger? We need to find a common denominator.

$\frac{5}{9} \bullet \frac{25}{25} = \frac{125}{225}$ $\frac{13}{25} \bullet \frac{9}{9} = \frac{117}{225}$

The yellow squares contain more area.

Alternate Solution:

David's 5 yellow squares have a total area of $\frac{5}{9} = .55...$ sq. in

John's 13 blue squares have an area of $\frac{13}{25} = .52$ sq. in

The yellow squares contain more area.

Alternate Solution:

David's square with the smaller yellow squares has 9 squares in it. John's square with the smaller blue squares has 25 squares in it. This is the LCM of 9 and 25 is 225.

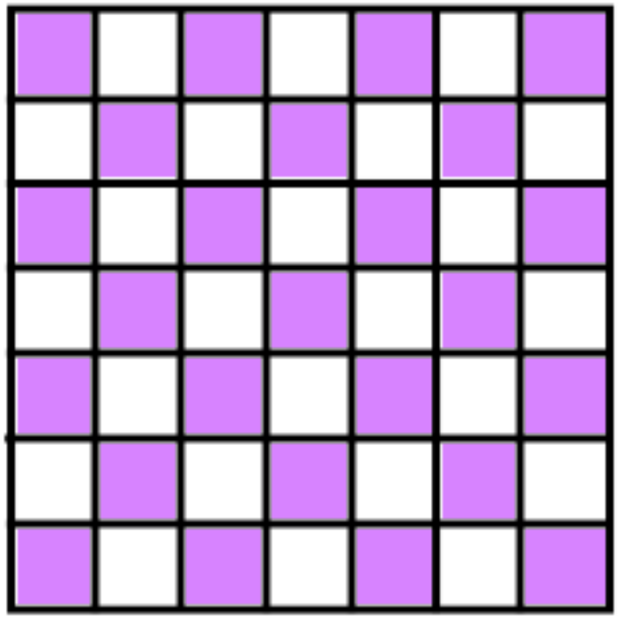
Let the area of both large squares have an area of 225 sq. units.

David's square has 9 smaller squares of equal area so each yellow square has an area of 25 sq. units. There are 5 of these yellow squares so the total area of the yellow squares is 125 sq. units.

John's square has 25 smaller squares of equal area so each blue square has an area of 9 sq. units. There are 13 of these blue squares so the total area of the blue squares is 117 Sq. units.
The yellow squares contain more area.

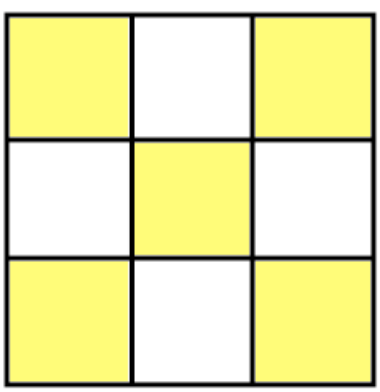
Tom divided his square into several smaller **purple** squares each of which were of equal size as shown below.

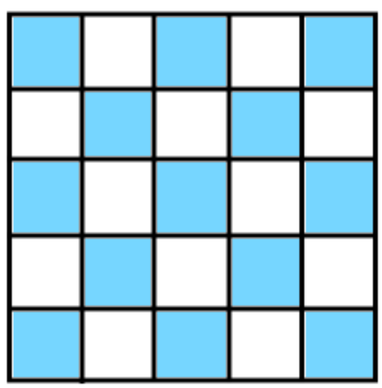
Tom's Square

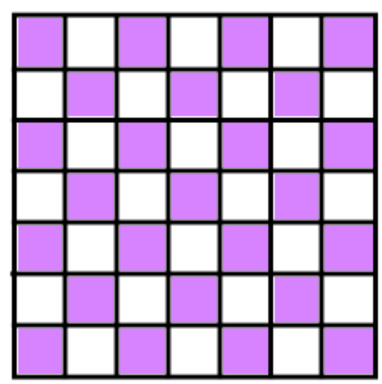


Tom's square with the smaller purple squares has 49 squares in it so the area of each purple square is 1/ 49 th of the total area. There are 25 of these purple squares so the total area of the purple squares is 25/ 49 th of a sq. in.

yellow area is $\frac{5}{9} = .55...$ sq. in **blue area is** $\frac{13}{25} = .52$ sq. in **purple area is** $\frac{25}{49} \approx .51$ sq. in





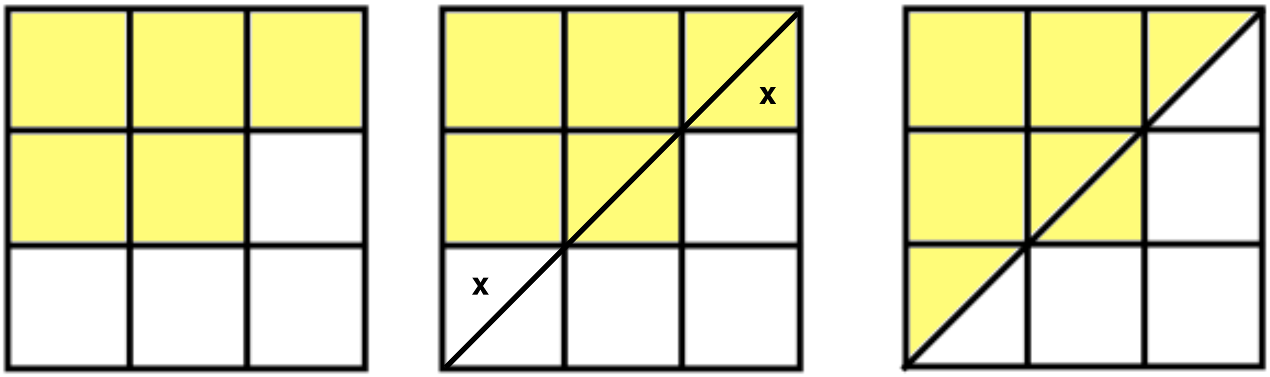


The total area of David's squares is more than John's which is more than Tom's.

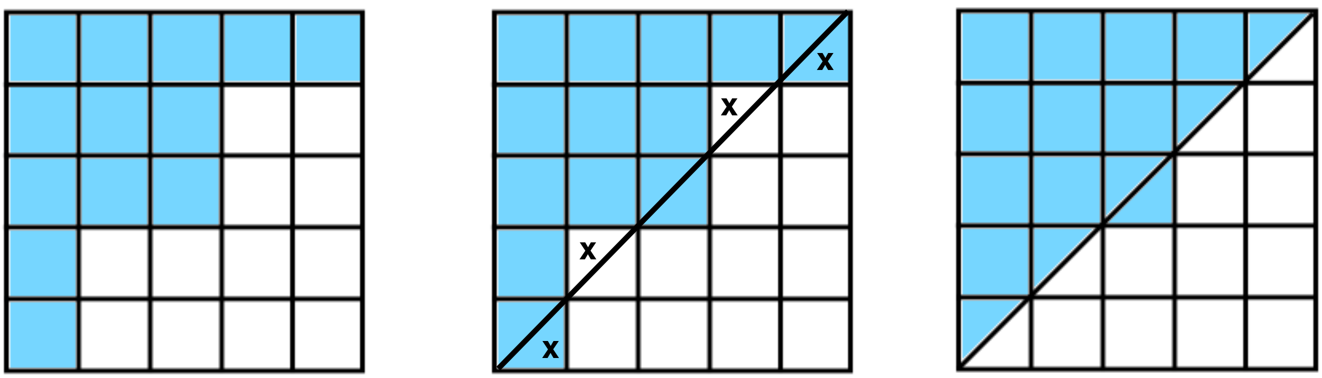
Number of divisions on each side of the square

D = number of squares per row	**N = 3**	**N = 5**	**N = 7**
Area of shaded squares =	$\frac{5}{9}$	$\frac{13}{25}$	$\frac{25}{49}$

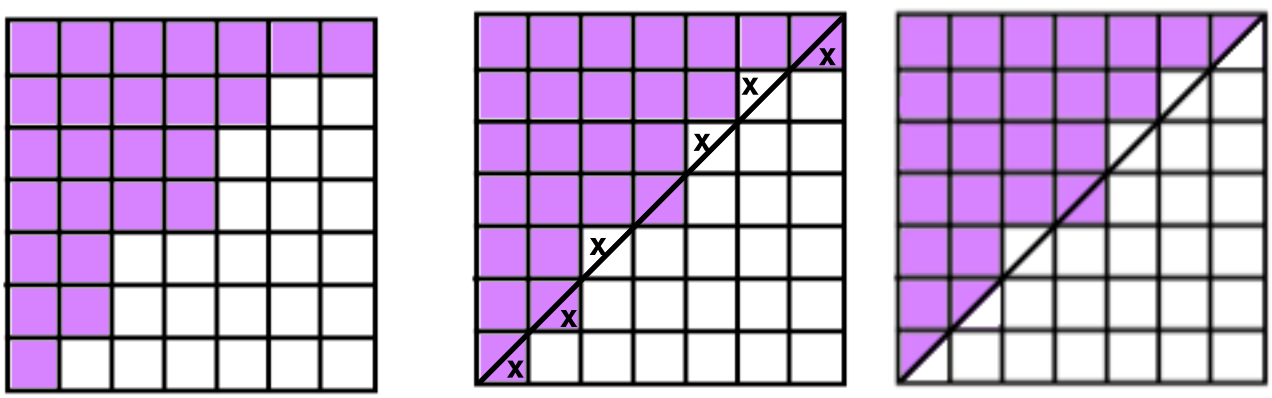
David has 5 yellow squares. Draw a diagonal to cut the square in half. Move the yellow area with an x to the white area area with an x. The diagonal cuts the square in half and there is 1/2 of a square extra. The yellow area is $\frac{1}{2}$ the area of the square $+\frac{1}{2}$



The blue area is $\frac{1}{2}$ the area of the square $+\frac{1}{2}$



The purple area is $\frac{1}{2}$ the area of the square $+\frac{1}{2}$



If there are n square in a row than the area of the square is n^2

and the area of the shade is $\frac{1}{2}$ the area of the square $+\frac{1}{2}$ **than**

the area of the shade is $\frac{1}{2}n^2+\frac{1}{2}$ **where n is the number of square in one row**

The first square was cut into a 3 by 3 grid. The second square was cut into a 5 by 5 grid. The third square was cut into a 7 by 7 grid.

What if I keep increasing the number of squares per row?

The number of squares on a row	**D = 3**	**D = 5**	**D = 7**	**D = 9**	**D = n**
Area of shaded squares =	$\frac{5}{9}$	$\frac{13}{25}$	$\frac{25}{49}$	$\frac{41}{81}$	$\frac{\frac{1}{2}n^2+\frac{1}{2}}{n^2}$

$$\frac{\frac{1}{2}n^2+\frac{1}{2}}{n^2}$$

$$=\frac{\frac{1}{2}\left(n^2+1\right)}{n^2}$$

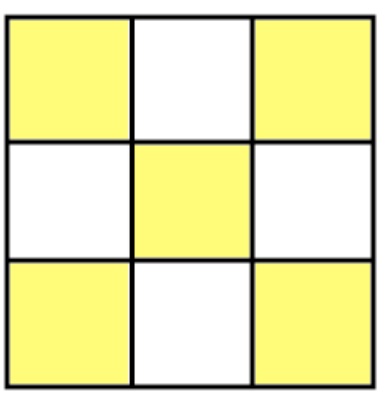
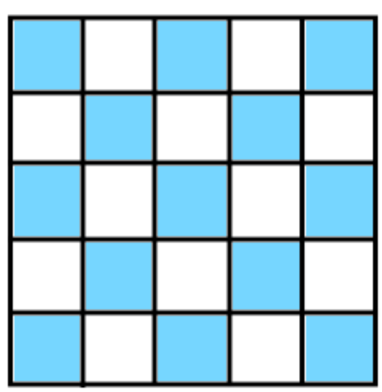
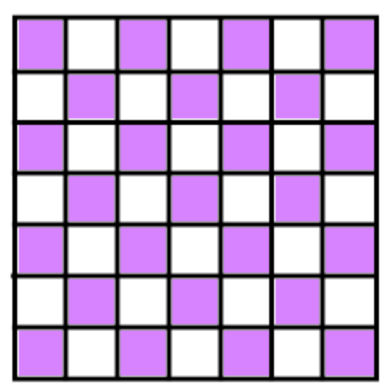
$$=\frac{n^2+1}{2n^2}$$

$$=\frac{n^2}{2n^2}+\frac{1}{2n^2}$$

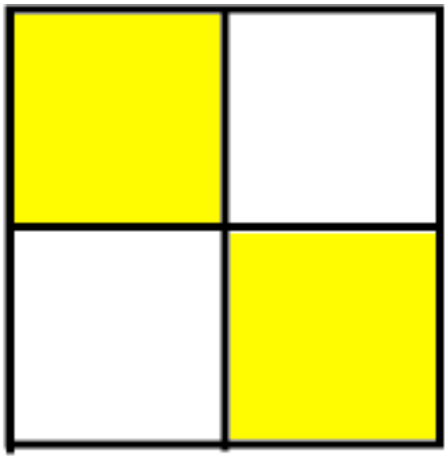
$$=\frac{1}{2}+\frac{1}{2n^2}$$

$$\lim_{n\to\infty} \frac{1}{2}+\frac{1}{2n^2}=\frac{1}{2}$$

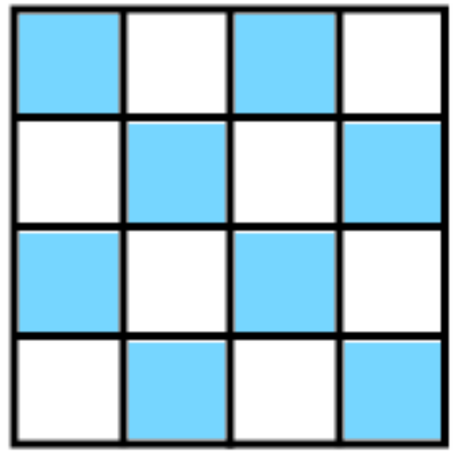
if I keep increasing the number of squares per row than the sum of the shaded squares will approach 1/ 2

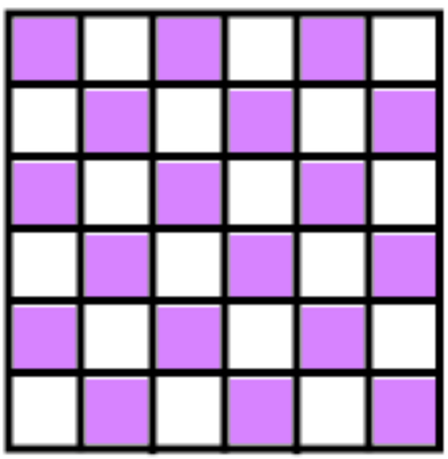
What if the square were cut into an EVEN number of square per row



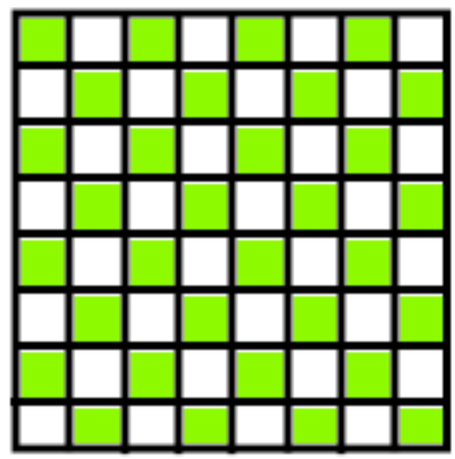
Area of the shaded squares $= \frac{2}{4} = \frac{1}{2}$



Area of the shaded squares $= \frac{8}{16} = \frac{1}{2}$



Area of the shaded squares $= \frac{18}{36} = \frac{1}{2}$

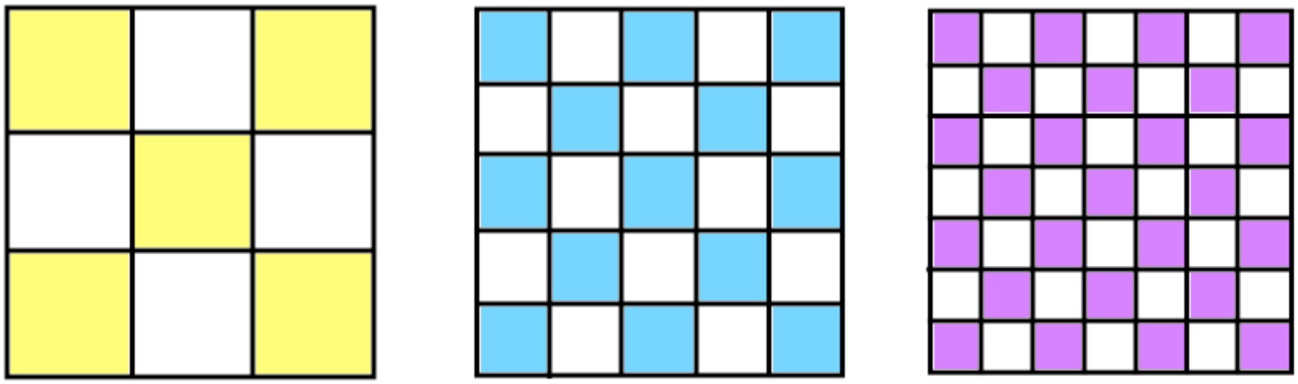


Area of the shaded squares $= \frac{32}{64} = \frac{1}{2}$

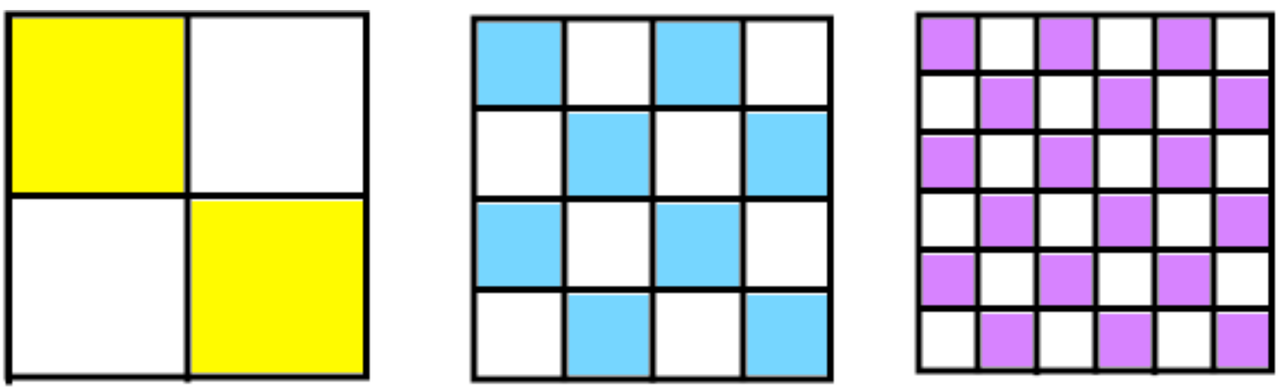
if I keep increasing the number of squares per row the sum of the shaded squares will always be 1/ 2

How does the area of the shade in the squares with an odd numbers of squares per row compare to the area of the shade squares in the squares with an odd numbers of squares?

If the large square is divided into rows with an odd number of squares per row the number of squares per row increases and the area of the shade approaches $\frac{1}{2}$



If the large square is divided into rows with an even number of squares per row the number of squares per row increases and the area of the shade is $\frac{1}{2}$



The area of the shade in the squares with an odd numbers of squares per row converges to the area of the shade in the squares with an even numbers of squares.