

# Sorting Squares

(Martin Gardner)

A student is given the large square below. They are asked to fold the paper forwards or backwards along any horizontal or vertical line. They are then asked to keep doing this until they have folded the packet into a single square packet. This will take from 4 to 6 folds.

You take the packet and cut along the 4 outside edges so that all the squares are separate. Deal out the 16 single squares. One of 2 things must happen. Either all the even numbered squares will be face up and all the odd numbered squares will be face down or visa versa depending on the way the paper was folded. This technique sorts out the even and odd squares.

1	2	3	4
8	7	<u>6</u>	5
<u>9</u>	10	11	12
<u>16</u>	15	14	13

## Preparation

Print out the square. Do not use heavy paper or card stock. Cut out the large square. You will need a good scissors to cut the final packet. You may have the student fold along all the vertical and horizontal line before you start the trick. That makes the packet come out more square.

## Presentation

A student is given the large square below. They are asked to fold the paper forwards or backwards along any horizontal or vertical line. They are then asked to keep doing this until they have the folded the packet into a single square. This will take from 4 to 6 folds.

You take the packet and cut along the 4 outside edges so that all the squares are separate. Be sure to cut far enough inside the edges to ensure the small squares are separate. There is space along the edges so that you can cut off the edges and still have the numbers show.

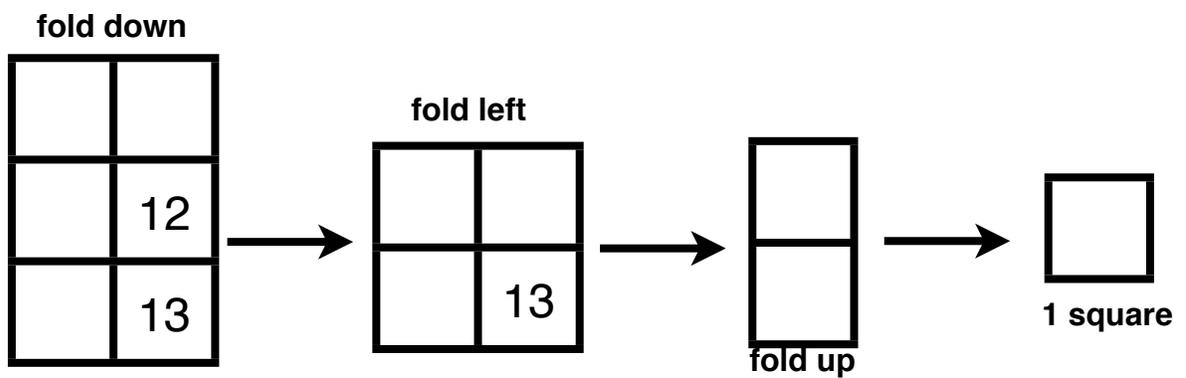
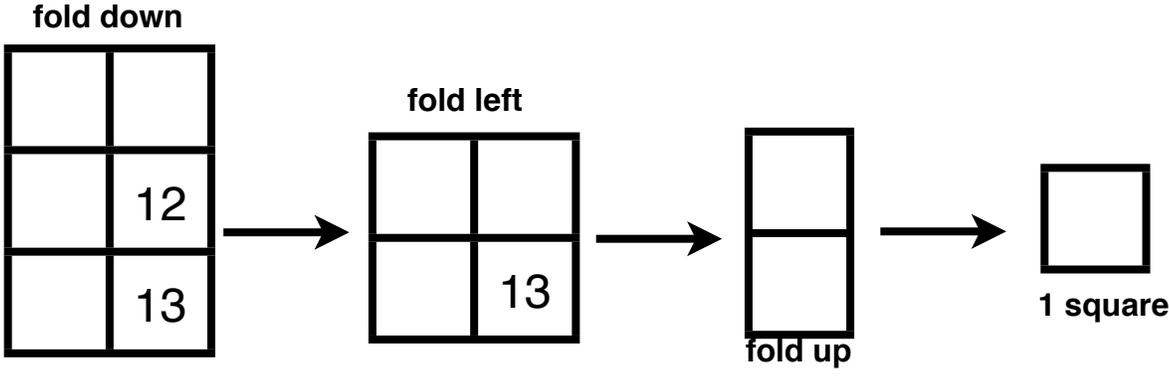
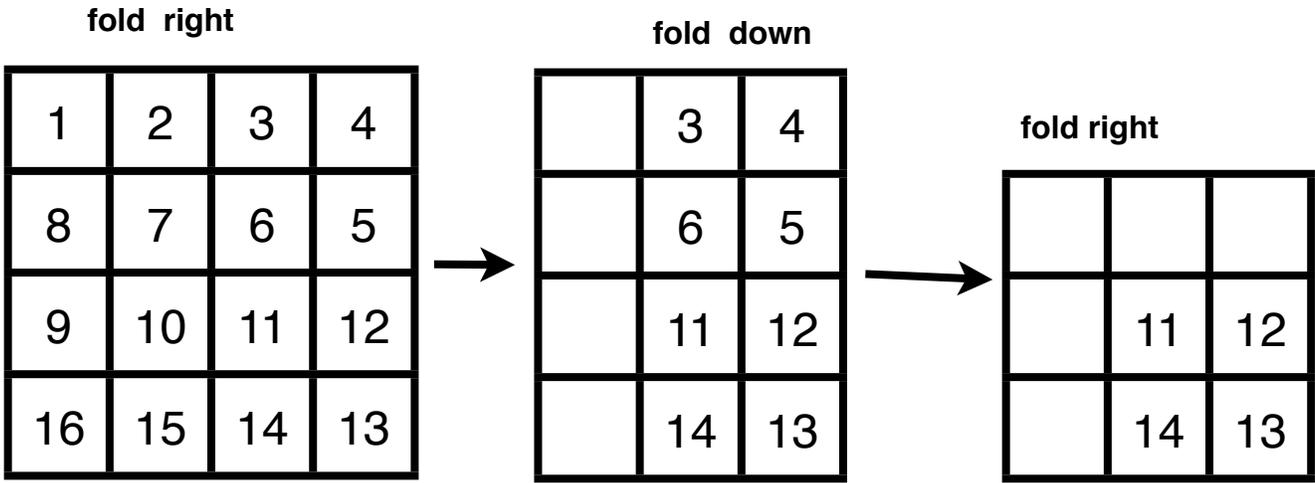
Deal out the 16 single squares . One of 2 things must happen. Either all the even numbered squares will be face up and all the odd numbered squares will be face down or visa versa depending on the way the paper was folded. This technique sorts out the even and odd squares.

**Deal 1:** Start dealing the separate squares between you and the student. Deal the numbered squares to them and the white faced squares to yourself. When you are done ask the student to spread out their squares You pick up your stack flip them over and spread out your squares. Let them see that you sorted the even numbered squares to one of you and the odd numbered squares to the other.

**Note:** Do not tell them in advance what you are going to do. Do not announce who will get the evens and who will get the odds because you do not know. When you see their cards just state that you were able to have them get all the evens (or odds) and you were able to get all the evens (or odds).

**Deal 2:** Start dealing the separate squares into 2 stacks. Numbers in one stack and white faces into the other. When you are done you pick up the numbers stack spread out the squares and show they are all even or odd. Flip over the other set of squares and spread them out. This stack will be the opposite of the first set.

One possible way to fold the square

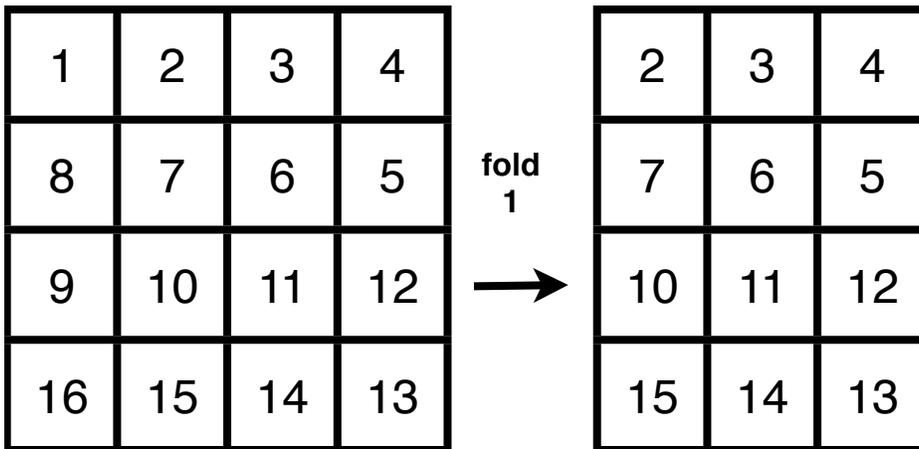


**How it works:**

The squares are numbered so the even numbered squares has odd squares on every side. The unique ordering ensures that any fold will always put an even and odd number facing opposite directions. When a fold is made the even squares involved in the fold end up being back to back with the odd squares they land on or they end up being face to face with the odd squares they land on. The even and odd squares that are on top of each other end up facing different directions. This is also true for each remaining folds. You could also state this rule in terms of the odd squares but the results are the same. A few attempts at folding the paper several ways will convince you the is true.

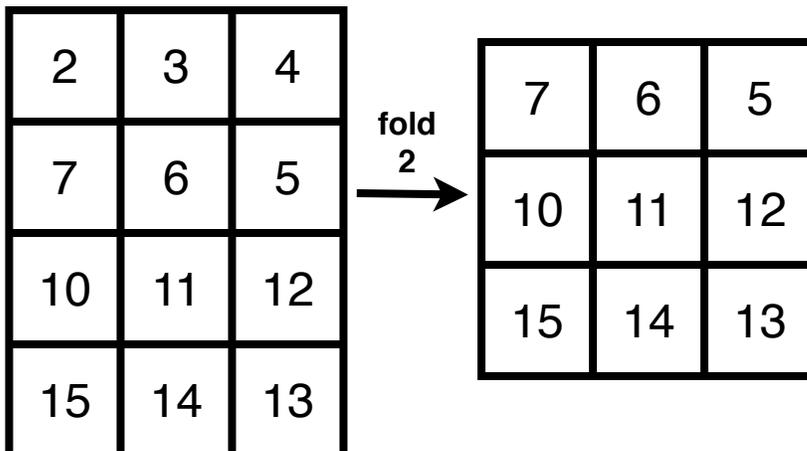
If may help if you take a square and try to follow the explanation .

**back  
fold  
right**

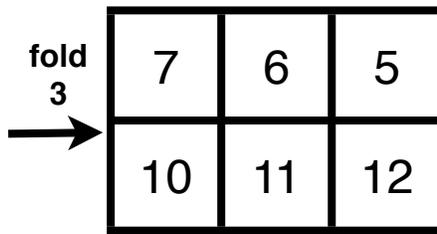
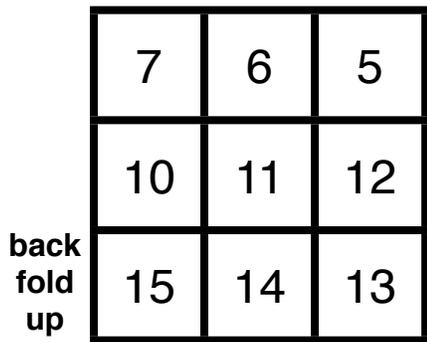


After the first fold the 1 and 2, the 8 and 7 , the 9 and 10 and the 15 and 16 are back to back so each pair of even - odds are facing different directions.

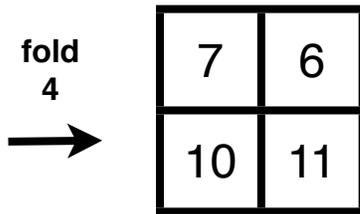
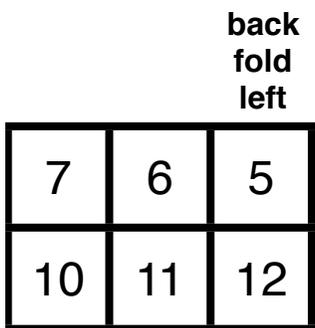
**back  
fold  
down**



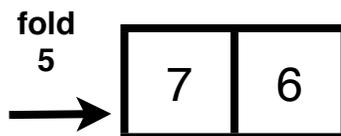
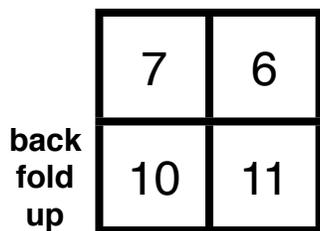
After the second fold the 1 and 7 are facing the same way and the 2 and 8 are facing the same way but the opposite way as the 1 and 7. Any new even odd squares that touch face opposite directions.



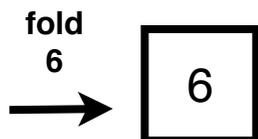
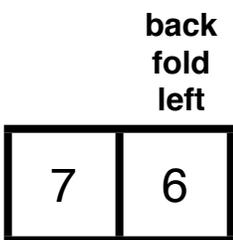
After the 3rd fold the 15 and 9 are facing the same way and the 10 and 16 are facing the same way but the opposite way as the 15 and 9. Any new even odd squares that touch face opposite directions.



After the 4th fold the 4 and 6 and the 3 and 5 are facing the same way but the opposite way as the 4 and 6. This is also true for the 11 and 13 and the 12 and 14.



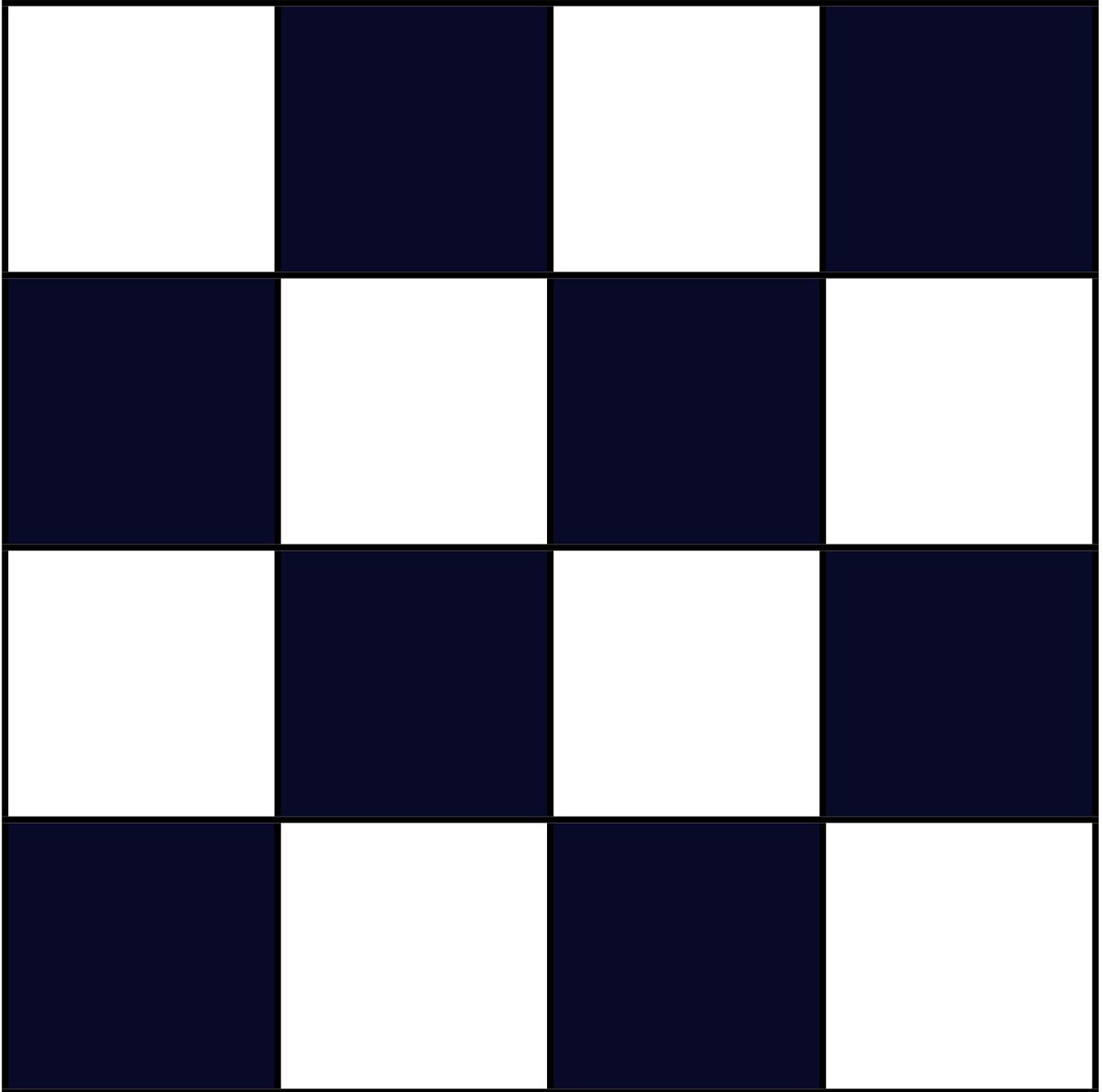
After the 4th fold the 2 and 10 and the 7 and 15 are facing the same way but the opposite way as the 2 and 10. This is also true for the 11 and 5 and the 12 and 6.



After the 6th fold the 11 and 9 are facing the same way as the other odds. All the evens face the same way but the opposite way as the odds.

## Chessboard Version

This version works just like the numbered square but the process sorts the squares into black and white squares.



## Card Trick Version

### Preparation.

Print out the square. Do not use heavy paper or card stock. Cut out the large square. You will need a good scissors to cut the final packet. You may have the student fold along all the vertical and horizontal line before you start the trick. That makes the packet come out more square. **You will also need a pen that writes in red and a pen that writes in black.**

### Presentation.

A student is reminded the a deck of cards has 52 cards. The 2 red suits are hearts and diamonds. The 2 black suits are clubs and spades. Each suit has 13 cards, A to 10 and the jack, queen and king. Ask the student to name their favorite RED card. When they state the card write the cards name down on an EVEN numbered square. Ask them to name any other 4 red cards. Write the names of those 4 cards down on an ODD numbered square.

Turn the square over. Ask another student to name his favorite BLACK card. When they state the card write the cards name down on an odd square. Ask them to name any other 4 BLACK cards. Write the names of those 4 cards down on an EVEN square.

They are asked to fold the paper forwards or backwards along any horizontal or vertical line. They are then asked to keep doing this until they have the folded the packet into a single square. This will take from 4 to 6 folds.

You take the packet and cut along the 4 outside edges so that all the squares are separate. Be sure to cut far enough inside the edges to ensure the small squares are separate. There is space along the edges so that you can cut off the edges and still have the numbers show.

Deal out the 16 single squares. One of 2 things must happen. Either all the even numbered squares will be face up and all the odd numbered squares will be face down or visa versa depending on the way the paper was folded. This technique sorts out the even and odd squares.

Start dealing the separate squares out on the desk. Deal the blank squares into a stack and the 5 squares with card names on the desk so each square can be seen. Say to the student who picked the black cards that the only black card showing is his favorite black card.

Put all the squares into one stack. Turn the stack over and deal out the squares just as you did the first time. This time you say to the student who picked the red cards that the only red card showing is his favorite red card.

### Example

Ask the student to name their favorite RED card. When they state the card write the cards name down on an EVEN numbered square. Ask them to name any other 4 red cards. Write the names of those 4 cards down on an ODD square.

The student says their **favorite RED card is the Ace of Hearts**. The student then names 4 other RED cards the **2D** , **7H**, **5D** and the **4H**.

1	2	3	4
8	7	<u>6</u>	5
<u>9</u>	10	11	12
16	15	14	13



**USE THE RED PEN**

Put the AH in an EVEN numbered square and the other RED cards in odd numbered squares

2D			
			7H
	AH	5D	
			4H

Turn the square over. Ask another student to name their favorite BLACK card. When they state the card write the cards name down on an ODD numbered square. Ask them to name any other 4 BLACK cards. Write the names of those 4 cards down on an EVEN square.

The second student says their **favorite BLACK card is the Ace of Clubs**. The student then names 4 other BLACK cards the **JC** , **8C**, **9S** and the **4S**.

1	2	3	4
8	7	<u>6</u>	5
<u>9</u>	10	11	12
<u>16</u>	15	14	13



**USE THE BLACK PEN**

Put the AH in an ODD numbered square and the other BLACK cards in EVEN numbered squares

			9S
	AC		
			4S
JC		8C	

Print pages 9 and 10 back to back if your printer supports back to back printing. If it does not support back to back printing then print 1 page and use a pen to draw in the lines on the opposite side of the square.

