

Over and Back

A student writes 4 numbers from 1 to 20 on the front of 4 cards.

Front of the cards

Student number A	Student number B	Student number C	Student number D
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The teacher writes 4 numbers of their own on the back of the 4 cards.

Back of the cards at the start

Teacher number $A + X$	Teacher number $B + X$	Teacher number $C + X$	Teacher number $D + X$
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Turn around so you cannot see the cards. Ask other students to watch to help ensure that the student correctly follows your instructions. Tell the student to take **any 2 of the papers** and turn them over. Take any 2 of the papers and turn them over again. Remind them that they can be any of the 4 papers. Ask them to do this one more time.

Remind them you cannot see the cards that are now face up and cannot know what the numbers on the cards are. Ask them to add up the 4 numbers that are now face up on the cards. Without turning around announce their total.

Over and Back

Preparation.

Cut out 4 small pieces of paper. I cut two 3 by 5 note cards in half. These cards are a good size and are not too flimsy to handle. Any size or shape of paper that will allow a student to write a large 2 digit number on it will work. If you plan on doing the trick a few times then cut out a few sets at the start.

Procedure:

Place one of the cards on the desk in front of the student. Ask them to write any number from 1 to 20 on the front of the card. Pick up the card, add 5 to their number, and write that number on the back of the card. Place the card **with their number facing up** back on the desk. Do the same thing for the other 3 cards.

Note: Try to write your number on the back of the paper in the same orientation as theirs on the front. Be sure to underline all 6's and 9's to avoid any confusion.

You now have 4 pieces of paper facing up with the student's 4 numbers on them. **Add the total of these 4 numbers in your head before you start the next state of the trick.**

Turn around so you cannot see the cards. Ask other students to watch to help ensure that the student correctly follows your instructions. Tell the student to take **any 2 of the papers** and turn them over. Take any 2 of the papers and turn them over again. Remind them that they can be any of the 4 papers. Ask them to do this one more time.

Remind them you cannot see the cards that are now face up and cannot know what the numbers on the cards are. Ask them to add up the 4 numbers that are now face up on the cards. Without turning around announce their total. **After 3 flips the total of the 4 cards now facing up will be the total of the 4 cards that were face up at the start of the trick plus 10.**

Note: Try to write your number on the back of the paper in the same orientation as theirs on the front. Be sure to underline all 6's and 9's to avoid any confusion.

Note: In many cases after 2 flips the total of the 4 cards now facing up will be the total of the 4 cards that were face up at the start of the trick plus 10 but that is not guaranteed. It is certain that **after an odd number of flips** the total of the 4 cards now facing up will be the total of the 4 cards that were face up at the start of the trick plus 10.

Example

Card 1 Front	Card 2 Front	Card 3 Front	Card 4 Front
The student writes 4 on the front	The student writes 11 on the front	The student writes 6 on the front	The student writes 12 on the front

Total of the
4 cards is
33

The student flips over Cards 1 and 3

Card 1 Back	Card 2 Front	Card 3 Back	Card 4 Front
you write $4 + 5$ 9 on the back	The student writes 11 on the front	you write $6 + 5$ 11 on the back	The student writes 12 on the front

The student flips over Cards 1 and 4

Card 1 Front	Card 2 Front	Card 3 Back	Card 4 Back
The student writes 4 on the front	The student writes 11 on the front	you write $6 + 5$ 11 on the back	you write $12 + 5$ 17 on the back

The student flips over Cards 2 and 3

Card 1 Front	Card 2 Front	Card 3 Back	Card 4 Back
The student writes 4 on the front	you write $11 + 5$ 16 on the back	The student writes 6 on the front	you write $12 + 5$ 17 on the back

After 3 flips the
total of the 4
cards is
 $33 + 10$
43

The first cards faces had a total of 33. After 3 flips the total of the 4 cards now facing up will be the total of the 4 cards that were face up at the start of the trick (33) plus 10.

Extensions:

The range of numbers that they can use to select their first 4 number.

Any range of numbers can be used. I used the numbers from 1 to 20 in my version. The size of sum of those 4 numbers seems reasonable for middle school students. Younger students may require numbers from 1 to 10 and older students may be able to use larger numbers. My purpose for doing this trick is to have the students discover the way it works. A set of small numbers meets that need for me.

The number that you add to their number on each card and put on the back of the card.

Any number can be used. The total of the 4 cards that are face up after an odd number of flips will be the total of the 4 cards that were face up at the start of the trick **plus twice the number you add to each student number**. I chose to add 5 to each student number. That means I will need to add 10 to the total of the 4 cards that were face up at the start of the trick. That seemed like an easy number to remember and add. If I repeated the trick for older students I would change that number each time I did the trick. Adding a 13 for instance would not be as obvious as adding 10. Students may not even notice each of your numbers are 13 larger than theirs.

Number of times the student flips the 2 pair of cards:

The number of times the student selects a pair of cards and flips them **MUST BE ODD**. I chose 3 flips for this trick. 1 flip would not be very impressive and 5 or 7 seem too many to keep student interest. The real value in this trick is trying to have the student understand why the number of flips needs to be an odd number. The proof of this is within their ability and is a great way to have them see what a proof may look like.

Theorem

The student picks 4 numbers for the front of the four cards. I will use A , B, C and D

The teacher adds the same number (X) to the student number for that card and puts that sum on the back of that card.

**The student picks any 3 cards and flips them over.
They do this a second and then a third time**

The total of the numbers facing up after 3 flips will always be $A + B + C + D + 2X$

Proof

Setup:

Front of the cards at the start

Student number A	Student number B	Student number C	Student number D
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At the start the total of the 4 cards is

$$A + B + C + D$$

If the student number is A the teacher writes $A + X$ on the back.

If the student number is B the teacher writes $B + X$ on the back

If the student number is C the teacher writes $C + X$ on the back

If the student number is D the teacher writes $D + X$ on the back

Back of the cards at the start

Teacher number $A + X$	Teacher number $B + X$	Teacher number $C + X$	Teacher number $D + X$
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When the student picks a student number to flip over the student number is exchanged for the teacher number which is the student number plus the number (X) the teacher added to it.

If the student flips the number A over they will see that $A + X$ is now on the front.

If the student flips the number B over they will see that $B + X$ is now on the front.

If the student flips the number C over they will see that $C + X$ is now on the front.

If the student flips the number D over they will see that $D + X$ is now on the front.

When the student picks a teacher number to flip over the teacher number is exchanged for the student number.

If the student flips the number $A + X$ over they will see that A is now on the front.

If the student flips the number $B + X$ over they will see that B is now on the front.

If the student flips the number $C + X$ over they will see that C is now on the front.

If the student flips the number $D + X$ over they will see that D is now on the front.

Front of the cards at the start

Student number A	Student number B	Student number C	Student number D
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At the start the total of the 4 cards is

$$A + B + C + D$$

Back of the cards at the start

Teacher number $A + X$	Teacher number $B + X$	Teacher number $C + X$	Teacher number $D + X$
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Flip 1

The student picks any 2 numbers and flips them over.

At the start all 4 numbers are student numbers so they must pick 2 student numbers to flip over.

When the student completes the first flip there will always be **2 of the student numbers**

and

2 of the teachers numbers facing up.

The student flips the B and D numbers and sees the following faces.

Student number A	Teacher number $B + X$	Student number C	Teacher number $D + X$
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After 1 flip the total of the 4 cards is always

$$A + B + C + D + 2X$$

Flip 2

There are 3 ways the student can select the cards for second flip

Case 1. On the second flip they flip 2 of the student numbers.

Case 2. On the second flip they flip 2 of the teachers numbers.

Case 3. On the second flip they flip 1 student and 1 teacher number.

Lets take each case one at a time and see what the final results will be for each Case.

**After the first flip there will always be
2 student numbers and 2 teacher numbers facing up.**

Student number A	Teacher number B + X	Student number C	Teacher number D + X
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After 1 flip the total of the 4 cards is
always
 $A + B + C + D + 2X$

Case 1. On the second flip they flip 2 of the student numbers

Lets look at Case 1 for the second flip and see where it leads.

Second flip: Pick any 2 teacher numbers and flip them over

Student number A	Teacher number B + X	Student number C	Teacher number D + X
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There are only 2 student numbers to select so the student flips them over. If Case 1 is used on the second flip there will **always be 4 teacher numbers facing up after the second flip**

After the second flip.

Teacher number A + X	Teacher number B + X	Teacher number C + X	Teacher number D + X
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In **Case 1** after 2 flips the total
of the 4 cards is always
 $A + B + C + D + 4X$

This means that Case 1 leads to a **third flip** where the only choice is to flip any 2 teachers numbers. This will always leave 2 student number and 2 teacher numbers facing up.

After the third flip.

Teacher number A + X	Student number B	Student number C	Teacher number D + X
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In **Case 1** after 3 flips the total
of the 4 cards is always
 $A + B + C + D + 2X$

This means that Case 1 leads to a **third flip** where the only choice is to flip any 2 teachers numbers. This will always leave 2 student number and 2 teacher numbers facing up.

No matter which teacher cards are flipped the total of 2 student number and 2 teacher numbers will always be $A + B + C + D + 2X$

**After the first flip there will always be
2 student numbers and 2 teacher numbers facing up.**

Student number A	Teacher number B + X	Student number C	Teacher number D + X
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After 1 flip the total of the 4 cards is
always
 $A + B + C + D + 2X$

Case 2. On the second flip they flip 2 of the teacher numbers

Lets look at Case 2 for the second flip and see where it leads.

Second flip: Pick any 2 teacher numbers and flip them over

Student number A	Teacher number B + X	Student number C	Teacher number D + X
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There are only 2 teacher numbers to select so the student flips them over. If Case 2 is used on the second flip there will **always be 4 student numbers facing up after the second flip**

After the second flip.

Student number A	Student number B	Student number C	Student number D
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In **Case 2** after 2 flips the total
of the 4 cards is always

$$A + B + C + D$$

This means that Case 2 leads to a **third flip** where the only choice is to flip any 2 student numbers. This will always leave 2 student number and 2 teacher numbers facing up.

After the third flip.

Teacher number A + X	Student number B	Student number C	Teacher number D + X
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In **Case 2** after 3 flips the total
of the 4 cards is always

$$A + B + C + D + 2X$$

No matter which student cards are flipped the total of 2 student number and 2 teacher numbers will always be $A + B + C + D + 2X$

**After the first flip there will always be
2 student numbers and 2 teacher numbers facing up.**

Student number A	Teacher number B + X	Student number C	Teacher number D + X
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After 1 flip the total of the 4 cards is
always
 $A + B + C + D + 2X$

Case 3. On the second flip they flip 1 student and 1 teacher number

Lets look at Case 3 for the second flip and see where it leads.

Second flip: Pick any 1 teacher number and 1 student number and flip them over

Student number A	Teacher number B + X	Student number C	Teacher number D + X
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Case 3. On the second flip they flip 1 student and 1 teacher number

If Case 3 is used on the second flip the student will flip 1 student number and get a teacher number in its place and flip 1 teacher card and get a student card in its place. No matter which numbers are selected the results will **always be 2 student number and 2 teacher numbers facing up after the second flip**

After the second flip.

Teacher number A + X	Student number B	Student number C	Teacher number D + X
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In **Case 3** after 2 flips the total of the 4 cards is always

$$A + B + C + D + 2X$$

This means that Case 2 leads to a third flip where the student will flip 1 student number and get a teacher number in its place and flip 1 teacher card and get a student card in its place. No matter which numbers are selected the results after the third flip will **always be 2 student number and 2 teacher numbers facing.**

After the third flip.

Teacher number A + X	Student number B	Student number C	Teacher number D + X
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In **Case 3** after 3 flips the total of the 4 cards is always

$$A + B + C + D + 2X$$

No matter which student cards are flipped the total of 2 student number and 2 teacher numbers will always be $A + B + C + D + 2X$

Conclusion

There is **ONLY 1 possible outcome after the first flip**

After the first flip there will always be 2 student numbers and 2 teacher numbers facing up and the total **will always be** $A + B + C + D + 2X$

There are **3 possible outcomes after the second flip**

There could be 4 teacher numbers facing up and the total will be $A + B + C + D + 4X$

or

There could be 4 student numbers facing up and the total will be $A + B + C + D$.

or

There could be 2 student numbers and 2 teacher numbers facing up and the total will always be $A + B + c + D + 2X$.

There is **ONLY 1 possible outcome after the third flip**

After the third flip there will always be 2 student numbers and 2 teacher numbers facing up and the **total will always be** $A + B + C + D + 2X$.

Theorem

The student picks 4 numbers for the front of the four cards. I will use A , B, C and D

The teacher adds the same number (X) to the student number for that card and puts that sum on the back of that card.

The student picks any 3 cards and flips them over.

They do this a second and then a third time

The total of the numbers facing up after 3 flips will always be $A + B + C + D + 2X$