

# Base Fibonacci Number Prediction Cards

## Base Fibonacci: Predict a Number from 1 to 88

1	4	6	9	12	14
17	19	22	25	27	30
33	35	38	40	43	46
48	51	53	56	59	61
64	67	69	72	74	77
80	82	85	88		

2	7	10	15	20	23
28	31	36	41	44	49
54	57	62	65	70	75
78	83	86			

3	4	11	12	16	17
24	25	32	33	37	38
45	46	50	51	58	59
66	67	71	72	79	80
87	88				

5	6	7	18	19	20
26	27	28	39	40	41
52	53	54	60	61	62
73	74	75	81	82	83

8	9	10	11	12	29
30	31	32	33	42	43
44	45	46	63	64	65
66	67	84	85	86	87
88					

13	14	15	16	17	18
19	20	47	48	49	50
51	52	53	54	68	69
70	71	72	73	74	75

21	22	23	24	25	26
27	28	29	30	31	32
33	76	77	78	79	80
81	82	83	84	85	86
87	88				

34	35	36	37	38	39
40	41	42	43	44	45
46	47	48	49	50	51
52	53	54			

55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88		

## Base Fibonacci Predict a Number from 1 to 88

**Procedure:** Cut out the 9 cards.

Ask a student to think of a number from 1 to 88 inclusive. You then hand the student the 9 cards. You ask the student to look at the cards and hand you all the cards that have their number on it. Tell them that their number may not be on all the cards. Be sure to ask them to look closely at the cards so they do not miss a number on one of the cards. After the cards with their number on them have been given back you announce their exact number!

**How it's done:**

Find the **smallest number** on each of the cards given to you. For these cards the smallest number is in the top left square. Add up those numbers. The total will be the number that they thought of.

**Example 1:**

The student picks 37 They hand you 2 cards.

The smallest number on one of the cards is 3

The smallest number on a second card is 34

Their number is  $34 + 3 = 37$

## Fibonacci Number Sequence

The number sequence 1 , 1 , 2 , 3 , 5 , 8 , 13 , 21 , 34 , 55 , 89 , 144 , 233 , 377 , 610 , .... is called the **Fibonacci Number Sequence**

The sequence starts by writing the first and second terms as 1's,      1 , 1  
you find the next term by adding the two terms just before it to get 2    1 , 1 , 2  
you find the next term by adding the two terms just before it to get 3    1 , 1 , 2 , 3  
you find the next term by adding the two terms just before it to get 5    1 , 1 , 2 , 3 , 5  
you find the next term by adding the two terms just before it to get 8    1 , 1 , 2 , 3 , 5 , 8  
you find the next term by adding the two terms just before it to get 13   1 , 1 , 2 , 3 , 5 , 8 , 13  
you find the next term by adding the two terms just before it to get 21   1 , 1 , 2 , 3 , 5 , 8 , 13 , 21  
continue adding the 2 numbers in the sequence to get the next term.

The number sequence appears in books by Indian mathematics first written in 200 BC. Indian mathematicians Pingala (200 BC), Virahanka (700 AD) and Gopaia (1135 AD) all refer to the numbers and develop real world applications concerning the numbers.

The first reference to the Fibonacci number sequence in Europe is in 1202 AD in the book Liber Abaci. The author of the book was Leonardo of Pisa who was also referred to as Fibonacci. It is at this point that the sequence starts being referred to as the Fibonacci sequence of numbers.

The number of applications to real world problems is amazing. Books relating to the appearance of these numbers in nature contain interesting relationships between how things in nature are designed based on this number sequence. A periodical of recent findings related to the sequence is printed yearly. After almost 2200 years mathematician are still discovering connections between these numbers and real world applications.

The cards in the previous pages used base 2, base 3 and base 4 to write base 10 numbers. One application of the Fibonacci number sequence involves using the numbers in the sequence as the base numbers for expressing base 10 digits. Instead of using 1, 2, 4, 8, 16, 32, 64 ... for the place values in base 2 we will use the Fibonacci numbers 1 , 2 , 3 , 5 , 8 , 13 , 21 , 34 , 55 , 89 , ... as the place values in base Fibonacci.

It has been proven that any base 10 number can be expressed in a unique way using the powers of 2 as the place values. We extend this by proving that the place vales could be the powers 3 or the powers of any whole number. Can we express any base 10 number using the Fibonacci numbers as place values. It turns out we can.

Zeckendorf's theorem states that every whole number can be represented uniquely as the sum of one or more distinct Fibonacci numbers in such a way that the sum does not include any two consecutive Fibonacci numbers. This theorem says that we can express any base 10 number using a unique set of Fibonacci numbers as place values where the digits of the fnumber in base fibonacci will be 1's or 0's.

# Writing a base 10 number in base Fibonacci

Zeckendorf's theorem states that every whole number can be represented using the place value values of 1, 2, 3, 5, 8, 13, 21 .... It also states that you can use only 1's and 0's as digits and that no two consecutive place values will have a 1's digit.

	<u>1 or 0</u>	<u>1 or 0</u>	<u>1 or 0</u>	<u>1 or 0</u>	<u>1 or 0</u>	<u>1 or 0</u>	<u>1 or 0</u>
place value	21	13	8	5	3	2	1

- Write the base 10 number as the sum of the fewest possible fibonacci numbers  
6 could be written as  $3 + 2 + 1$  but it written as  $5 + 1$
- Place a 1 in each place value for the numbers used in the sum and a 0 in the other place values as place holders.

	<u>1 or 0</u>	<u>1 or 0</u>	<u>1 or 0</u>	<u>1 or 0</u>	<u>1 or 0</u>	<u>1 or 0</u>	<u>1 or 0</u>
place value	21	13	8	5	3	2	1

## Example 1 Convert 12 base 10 to base fibonacci

$$12 \text{ base } 10 = 8 + 3 + 1$$

digit	<u>1</u>	<u>0</u>	<u>1</u>	<u>0</u>	<u>1</u>
place value	8	5	3	2	1

$$12 \text{ base } 10 = 10101 \text{ base fib}$$

## Example 2 Convert 18 base 10 to base fibonacci

$$18 \text{ base } 10 = 13 + 5$$

digit	<u>1</u>	<u>0</u>	<u>1</u>	<u>0</u>	<u>0</u>	<u>0</u>
place value	13	8	5	3	2	1

$$18 \text{ base } 10 = 101000 \text{ base fib}$$

## Example 3 Convert 34 base 10 to base fibonacci

$$34 \text{ base } 10 = 15 + 13 + 5 + 1$$

digit	<u>1</u>	<u>1</u>	<u>0</u>	<u>1</u>	<u>0</u>	<u>0</u>	<u>1</u>
place value	15	13	8	5	3	2	1

$$34 \text{ base } 10 = 1101001 \text{ base fib}$$

### Base Fibonacci digits for the numbers 1 to 30 base 10

Base 10	base 10 as a sum of Fibonacci numbers	card 55	card 34	card 21	card 13	card 8	card 5	card 3	card 2	card 1
1	1	0	0	0	0	0	0	0	0	1
2	2	0	0	0	0	0	0	0	1	0
3	3	0	0	0	0	0	0	1	0	0
4	3 + 1	0	0	0	0	0	0	1	0	1
5	5	0	0	0	0	0	1	0	0	0
6	5 + 1	0	0	0	0	0	1	0	0	1
7	5 + 2	0	0	0	0	0	1	0	1	0
8	8	0	0	0	0	1	0	0	0	0
9	8 + 1	0	0	0	0	1	0	0	0	1
10	8 + 2	0	0	0	0	1	0	0	1	0
11	8 + 3	0	0	0	0	1	0	1	0	0
12	8 + 3 + 1	0	0	0	0	1	0	1	0	1
13	13	0	0	0	1	0	0	0	0	0
14	13 + 1	0	0	0	1	0	0	0	0	1
15	13 + 2	0	0	0	1	0	0	0	1	0
16	13 + 3	0	0	0	1	0	0	1	0	0
17	13 + 3 + 1	0	0	0	1	0	0	1	0	1
18	13 + 5	0	0	0	1	0	0	0	0	0
19	13 + 5 + 1	0	0	0	1	0	1	0	0	1
20	13 + 5 + 2	0	0	0	1	0	1	0	1	0
21	21	0	0	1	0	0	0	0	0	0
22	21 + 1	0	0	1	0	0	0	0	0	1
23	21 + 2	0	0	1	0	0	0	0	1	0
24	21 + 3	0	0	1	0	0	0	1	0	0
25	21 + 3 + 1	0	0	1	0	0	0	1	0	1
26	21 + 5	0	0	1	0	0	1	0	0	0
27	21 + 5 + 1	0	0	1	0	0	1	0	0	1
28	21 + 5 + 2	0	0	1	0	0	1	0	1	0
29	21 + 8	0	0	1	0	1	0	0	0	0
30	21 + 8 + 1	0	0	1	0	1	0	0	0	1

### Base Fibonacci digits for the numbers 31 to 60 base 10

Base 10	base 10 as a sum of Fibonacci numbers	card 55	card 34	card 21	card 13	card 8	card 5	card 3	card 2	card 1
31	$21 + 8 + 2$	0	0	1	0	1	0	0	1	0
32	$21 + 8 + 3$	0	0	1	0	1	0	1	0	0
33	$21 + 8 + 3 + 1$	0	0	1	0	1	0	1	0	1
34	34	0	1	0	0	0	0	0	0	0
35	$34 + 1$	0	1	0	0	0	0	0	0	1
36	$34 + 2$	0	1	0	0	0	0	0	1	0
37	$34 + 3$	0	1	0	0	0	0	1	0	0
38	$34 + 3 + 1$	0	1	0	0	0	0	0	0	1
39	$34 + 5$	0	1	0	0	0	1	0	0	0
40	$34 + 5 + 1$	0	1	0	0	0	1	0	0	1
41	$34 + 5 + 2$	0	1	0	0	0	1	0	1	0
42	$34 + 8$	0	1	0	1	0	0	0	0	0
43	$34 + 8 + 1$	0	1	0	1	0	0	0	0	1
44	$34 + 8 + 2$	0	1	0	1	0	0	0	1	0
45	$34 + 8 + 3$	0	1	0	1	0	0	1	0	0
46	$34 + 8 + 3 + 1$	0	1	0	1	1	0	1	0	1
47	$34 + 13$	0	1	0	0	1	0	0	0	0
48	$34 + 13 + 1$	0	1	0	0	1	0	0	0	1
49	$34 + 13 + 2$	0	1	0	0	1	0	0	1	0
50	$34 + 13 + 3$	0	1	0	0	1	0	1	0	0
51	$34 + 13 + 3 + 1$	0	1	0	0	1	0	1	0	1
52	$34 + 13 + 5$	0	1	0	0	1	1	0	0	0
53	$34 + 13 + 5 + 1$	0	1	0	0	1	1	0	0	1
54	$34 + 13 + 5 + 2$	0	1	0	0	1	1	0	1	0
55	55	1	0	0	0	1	0	0	0	0
56	$55 + 1$	1	0	0	0	1	0	0	0	1
57	$55 + 2$	1	0	0	0	1	0	0	1	0
58	$55 + 3$	1	0	0	0	1	0	1	0	0
59	$55 + 3 + 1$	1	0	0	0	1	0	1	0	1
60	$55 + 5$	1	0	0	0	1	1	0	0	0



### Base Fibonacci digits for the numbers 61 to 88 base 10

Base 10	base 10 as a sum of Fibonacci numbers	card 55	card 34	card 21	card 13	card 8	card 5	card 3	card 2	card 1
61	$55 + 5 + 1$	1	0	0	0	0	1	0	0	1
62	$55 + 5 + 2$	1	0	0	0	0	1	0	1	0
63	$55 + 8$	1	0	0	0	1	0	0	0	0
64	$55 + 8 + 1$	1	0	0	0	1	0	0	0	1
65	$55 + 8 + 2$	1	0	0	0	1	0	0	1	0
66	$55 + 8 + 3$	1	0	0	0	1	0	1	0	0
67	$55 + 8 + 3 + 1$	1	0	0	0	0	0	1	0	1
68	$55 + 13$	1	0	0	1	0	0	0	0	0
69	$55 + 13 + 1$	1	0	0	1	0	0	0	0	1
70	$55 + 13 + 2$	1	0	0	1	0	0	0	1	0
71	$55 + 13 + 3$	1	0	0	1	0	0	1	0	0
72	$55 + 13 + 3 + 1$	1	0	0	1	0	0	1	0	1
73	$55 + 13 + 5$	1	0	0	1	0	1	0	0	0
74	$55 + 13 + 5 + 1$	1	0	0	1	0	1	0	0	1
75	$55 + 13 + 5 + 2$	1	0	0	1	0	1	0	1	0
76	$55 + 21$	1	0	1	0	0	0	0	0	0
77	$55 + 21 + 1$	1	0	1	0	0	0	0	0	1
78	$55 + 21 + 2$	1	0	1	0	0	0	0	1	0
79	$55 + 21 + 3$	1	0	1	0	0	0	1	0	0
80	$55 + 21 + 3 + 1$	1	0	1	0	0	0	1	0	1
81	$55 + 21 + 5$	1	0	1	0	0	1	0	0	0
82	$55 + 21 + 5 + 1$	1	0	1	0	0	1	0	0	1
83	$55 + 21 + 5 + 2$	1	0	1	0	0	1	0	1	0
84	$55 + 21 + 8$	1	0	1	0	1	0	0	0	0
85	$55 + 21 + 8 + 1$	1	0	1	0	1	0	0	0	1
86	$55 + 21 + 8 + 2$	1	0	1	0	1	0	0	1	0
87	$55 + 21 + 8 + 3$	1	0	1	0	1	0	1	0	0
88	$55 + 21 + 8 + 3 + 1$	1	0	1	0	1	0	1	0	1