Next Door Neighbors (Gardner)

Place the numbers 1 to 8 in the boxes below.   Consecutive numbers **CANNOT** be in adjacent boxes. Adjacent boxes are boxes connected by a dashed line.

1 , 2 , 3 , 4 , 5 , 6 , 7 , 8
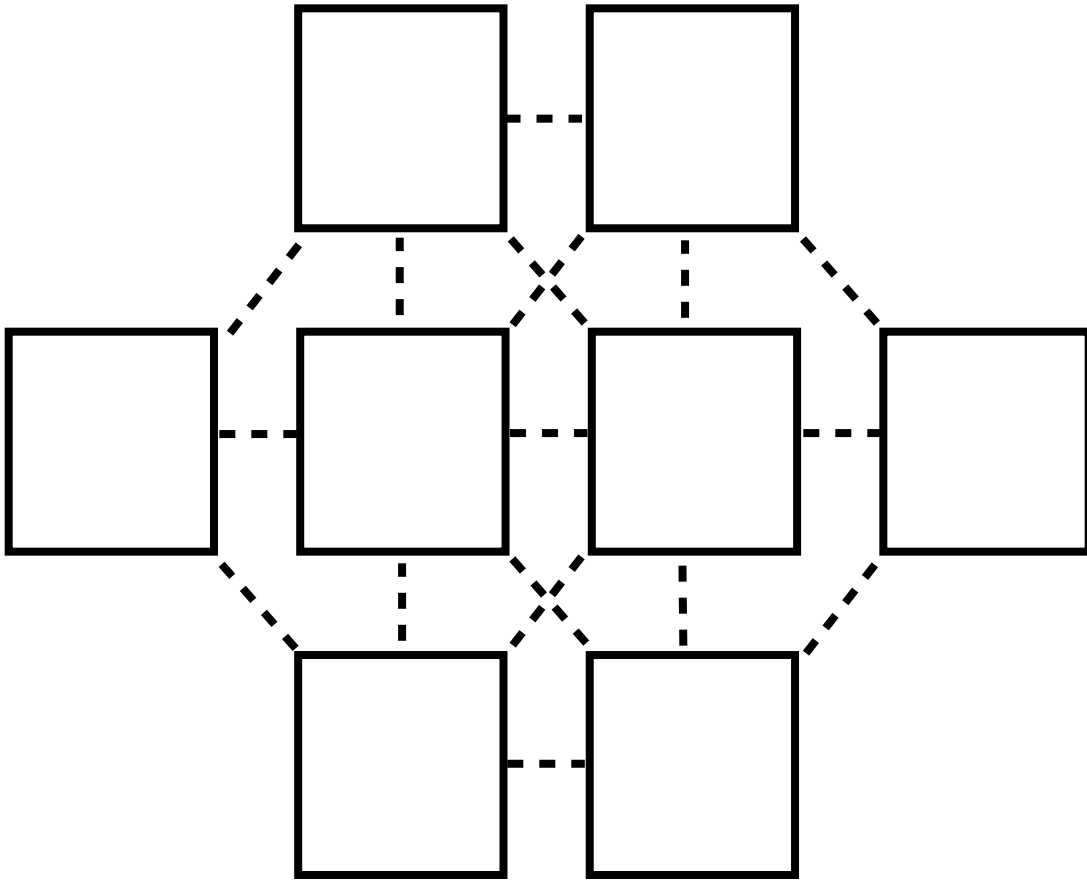
Worksheet
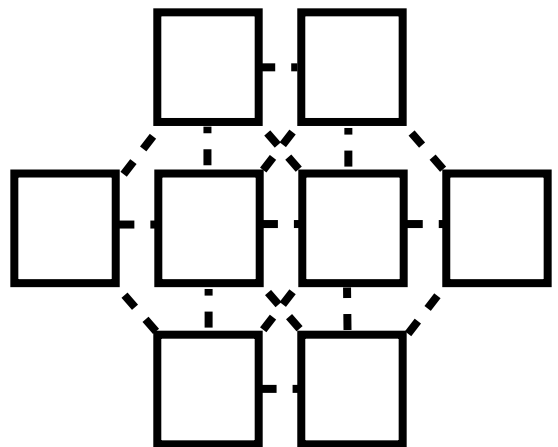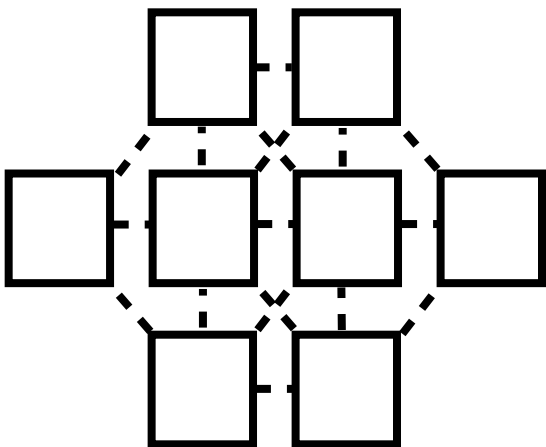
One Possible Solution

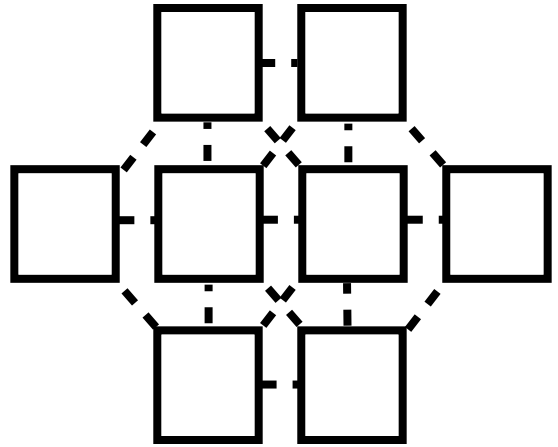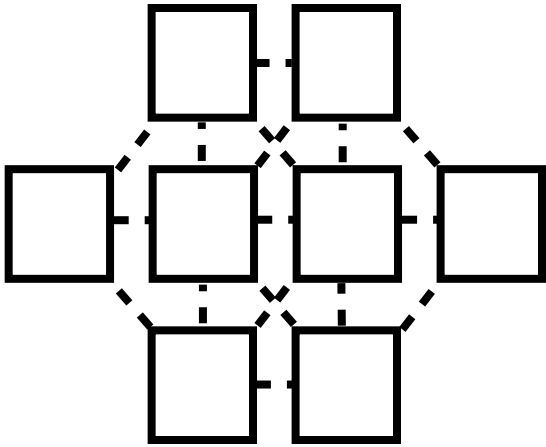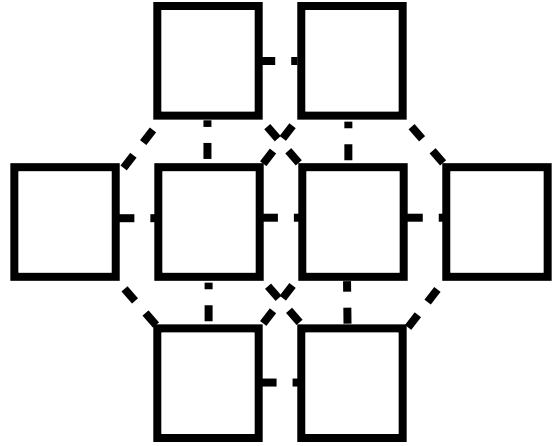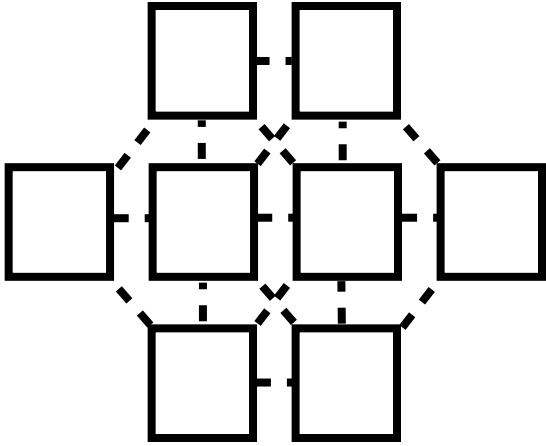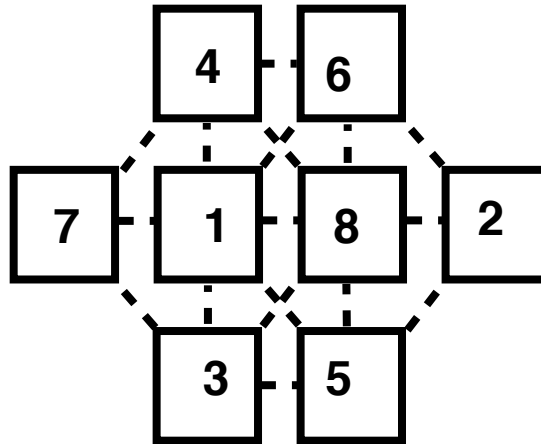This puzzle was first published by Martin Gardner in the 1960's. It was also presented on a TV show at the time to contrast how a computer would just search through the over 40,00 possible combinations until he solution was fond versus how a human using logic could find the solution using far fewer combinations. The use of logic to reduce the possible combinations also leads to being able to determine and describe all the possible solutions.

This is a great puzzle for students to work on. They can find the solution in many cases by working with the sample blank boxes. The real strength is in trying to work with them to develop the logic for the solution. As they work towards a generalized description of a solution they lean the wording and order used in logical proofs.

Do not give in to the temptation to find the solution and then move on. Help your students stay with the problem until THEY can talk (or write) through the complete description of the solution.

Finding the solution

Lets label the boxes with letter as shown below.

```
        ┌───┐ ┌───┐
        │ E │─│ F │
        └───┘ └───┘
          │     │
┌───┐ ┌───┐ ┌───┐ ┌───┐
│ C │─│ A │─│ B │─│ D │
└───┘ └───┘ └───┘ └───┘
          │     │
        ┌───┐ ┌───┐
        │ G │─│ H │
        └───┘ └───┘
```
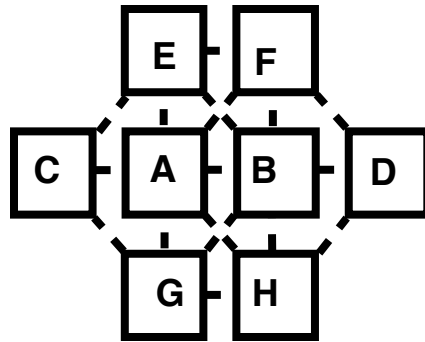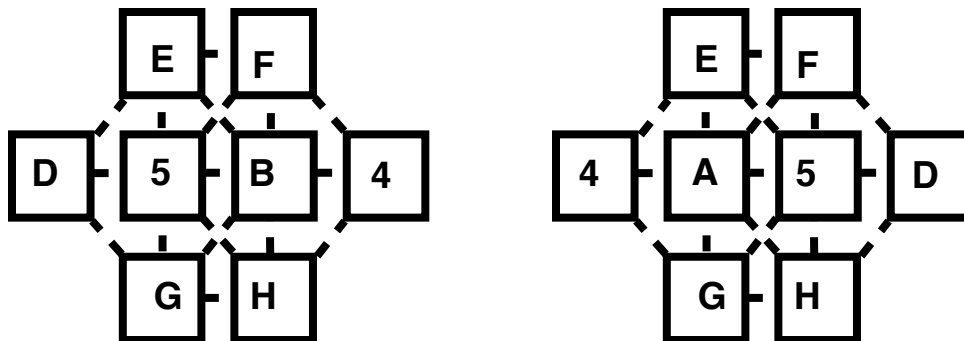
First note that Box A and Box B have 6 boxes that are adjacent to them. This is the key to limiting the possible numbers that can go into Box A.   If you out a number on Box A,

Next note that 1 and 8 have only one number adjacent to them while 2,3,4,5,6 and 7 each have 2 numbers adjacent to them. For example 1 only has the number 2 as an adjacent number while 3 has the numbers 2 and 4 as adjacent numbers.

Claim: None of the number 2,3,4,5,6 and 7 can be put in Box A or Box B.

If you select any of the numbers 2,3,4,5,6 and 7 and put it in box A then you need to place the numbers number on less and one more than that number in boxes that are not adjacent to Box A. That is not possible to do. Lets try that with 5 as an example. If 5 is put in Box A then 4 cannot go in any box except Box D as all the other boxes are adjacent to 5. That means that the number 6 must go in one of the remaining boxes but they are all adjacent to 5 so 6 cannot be placed there. The same issue happens if you put the 5 in Box B

```
        ┌───┐ ┌───┐              ┌───┐ ┌───┐
        │ E │─│ F │              │ E │─│ F │
        └───┘ └───┘              └───┘ └───┘
          │     │                  │     │
┌───┐ ┌───┐ ┌───┐ ┌───┐  ┌───┐ ┌───┐ ┌───┐ ┌───┐
│ D │─│ 5 │─│ B │─│ 4 │  │ 4 │─│ A │─│ 5 │─│ D │
└───┘ └───┘ └───┘ └───┘  └───┘ └───┘ └───┘ └───┘
          │     │                  │     │
        ┌───┐ ┌───┐              ┌───┐ ┌───┐
        │ G │─│ H │              │ G │─│ H │
        └───┘ └───┘              └───┘ └───┘
```

Conclusion: The numbers 1 and 8 must go in Boxes A and B in either order

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```
    E   F              E   F
C   1   8   D      C   8   1   D
    G   H              G   H
```
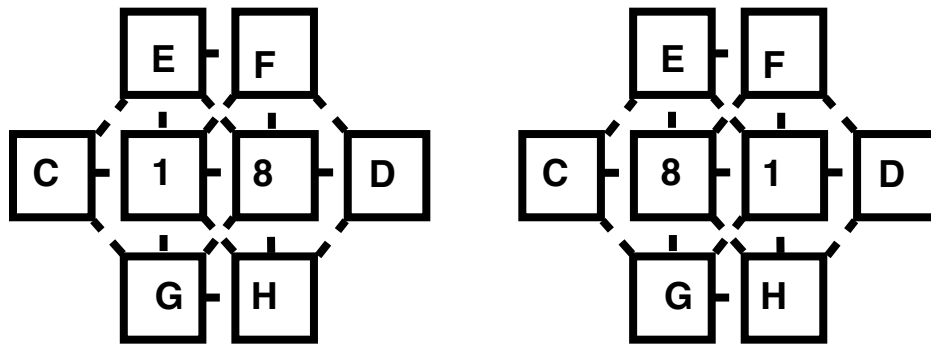
Next conclusions:

1. The only box not adjacent to the number 1 is the box at the far end of the same row 1 is on. 2 is the only number adjacent to 1 so 2 must go is the box at the far end of the same row 1 is on.

2. The only box not adjacent to the number 8 is the box at the far end of the same row 8 is on. 7 is the only number adjacent to 8 so 7 must go is the box at the far end of the same row 8 is on.
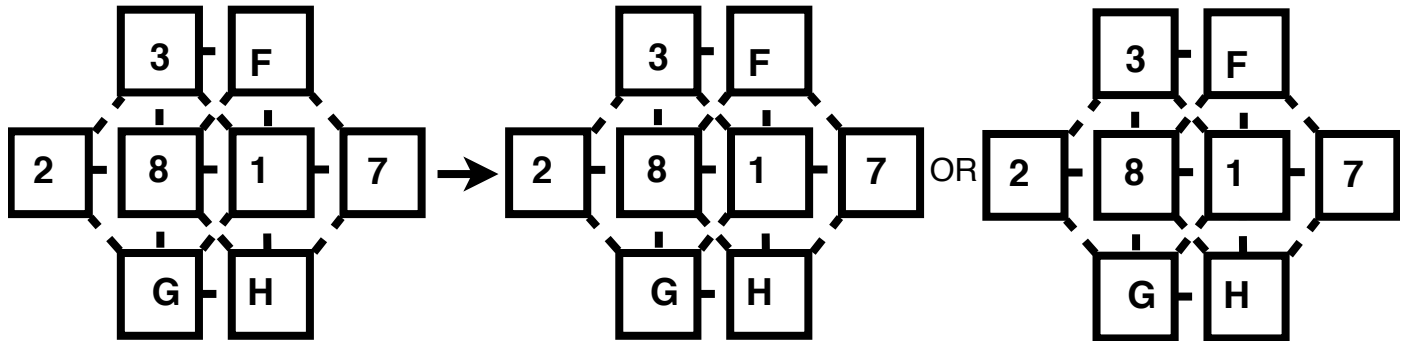
The only 2 possible arrangements the numbers 1, 2, 7 and 8 can be placed are shown below.

```
      E   F                  E   F
7   1   8   2          2   8   1   7
      G   H                  G   H
```
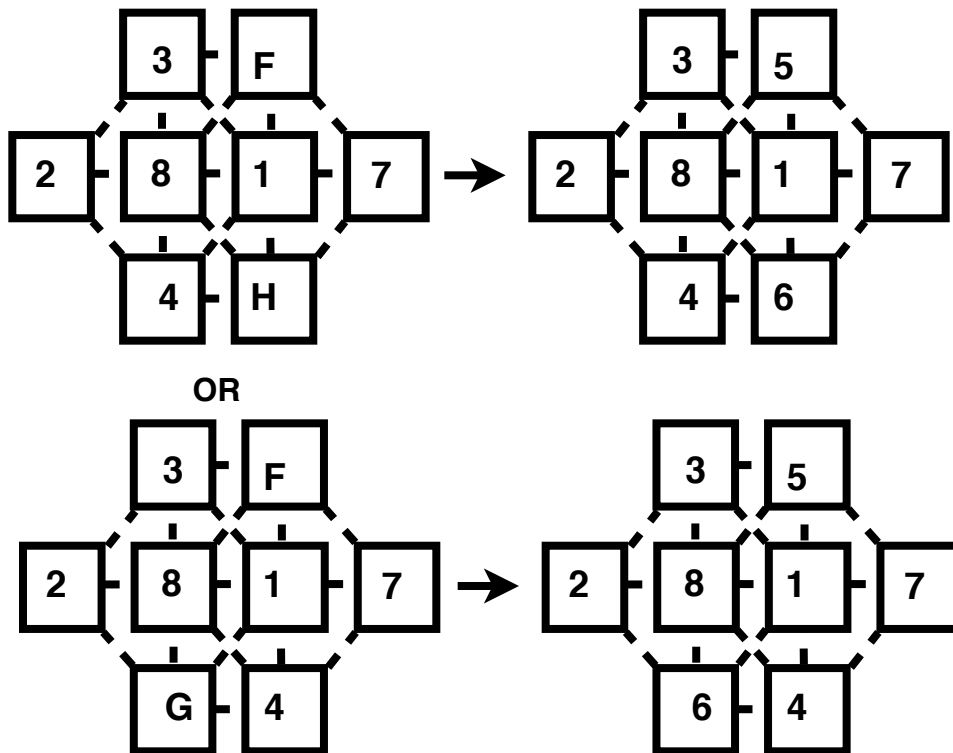
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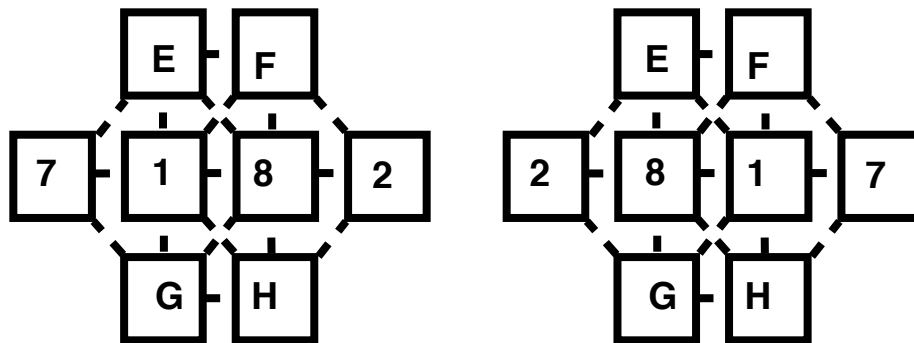
Next observations.

1. Any of the remaining numbers 3 ,4 5 or 6 could go in box E. The number adjacent to it must go in the bottom row in either box G or box H.

Example

```
    3   F              3   F                3   F
  2 8 1 7      →     2 8 1 7       OR     2 8 1 7
    G   H              G   H                G   H
```
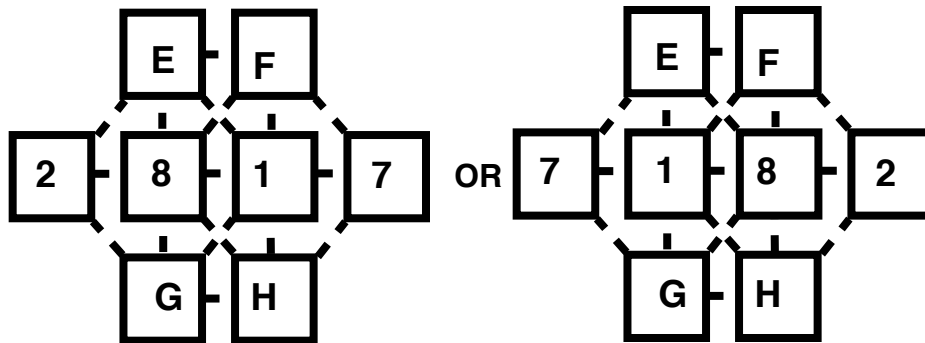
The last 2 numbers will be placed in the 2 boxes remaining so they are not adjacent to the numbers already placed in the adjacent boxes.

```
    3   F               3   5
  2 8 1 7       →     2 8 1 7
    4   H               4   6
```

OR

```
    3   F               3   5
  2 8 1 7       →     2 8 1 7
    G   4               6   4
```

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Final Solution:

1. The only 2 possible arrangements the numbers 1, 2, 7 and 8 can be placed are shown below.

E	F	
2	8 1	7
G	H	

OR

E	F	
7	1 8	2
G	H	

3. The last 4 number have the following possible placements; The numbers 3and 5 must be paired on the top or bottom in either order and the 6 and 6 must be placed in either order in the remaining boxes.

5	3	
2	8 1	7
4	6	

3	5	
2	8 1	7
6	4	

5	3	
2	8 1	7
4	6	

5	3	
2	8 1	7
6	4	