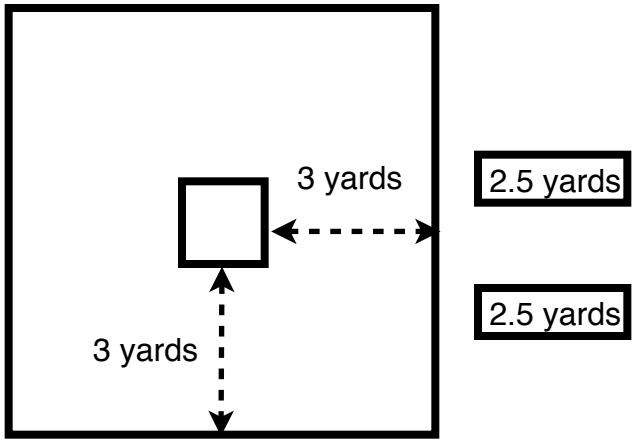
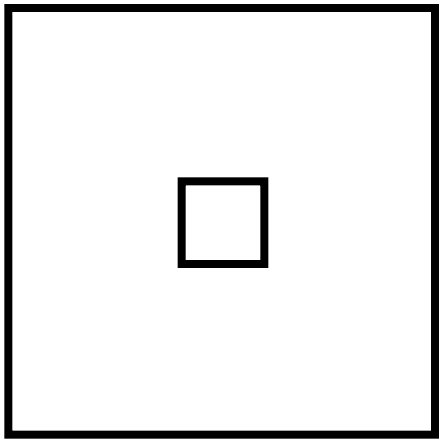
Get to The Center

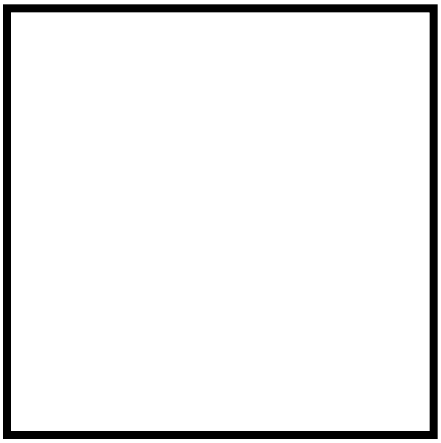
You are on the bank of a square pond and can't swim. There is a square island in the middle of the pond. The island is 3 yards from all sides of the pond as shown in the figure below. You need to use the 2 boards shown to create a way to get to the island.

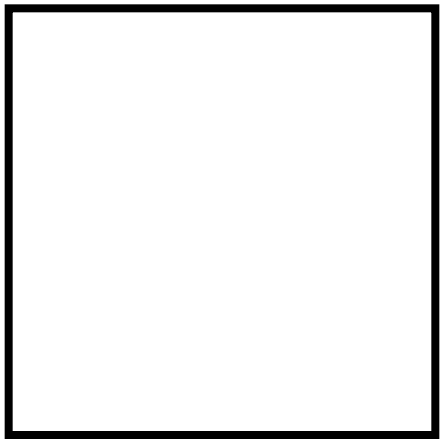
Draw your solution on the figure shown at the right.

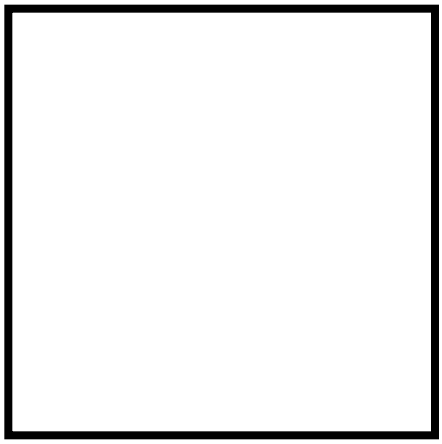


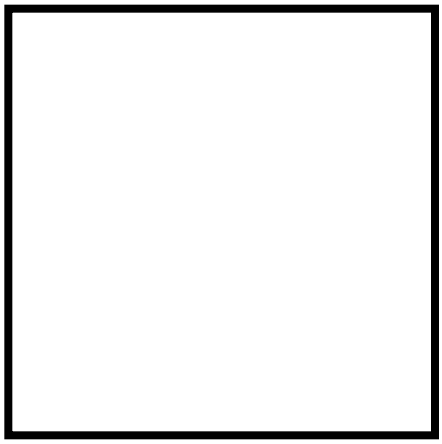


Extra copies for you to try different solutions.





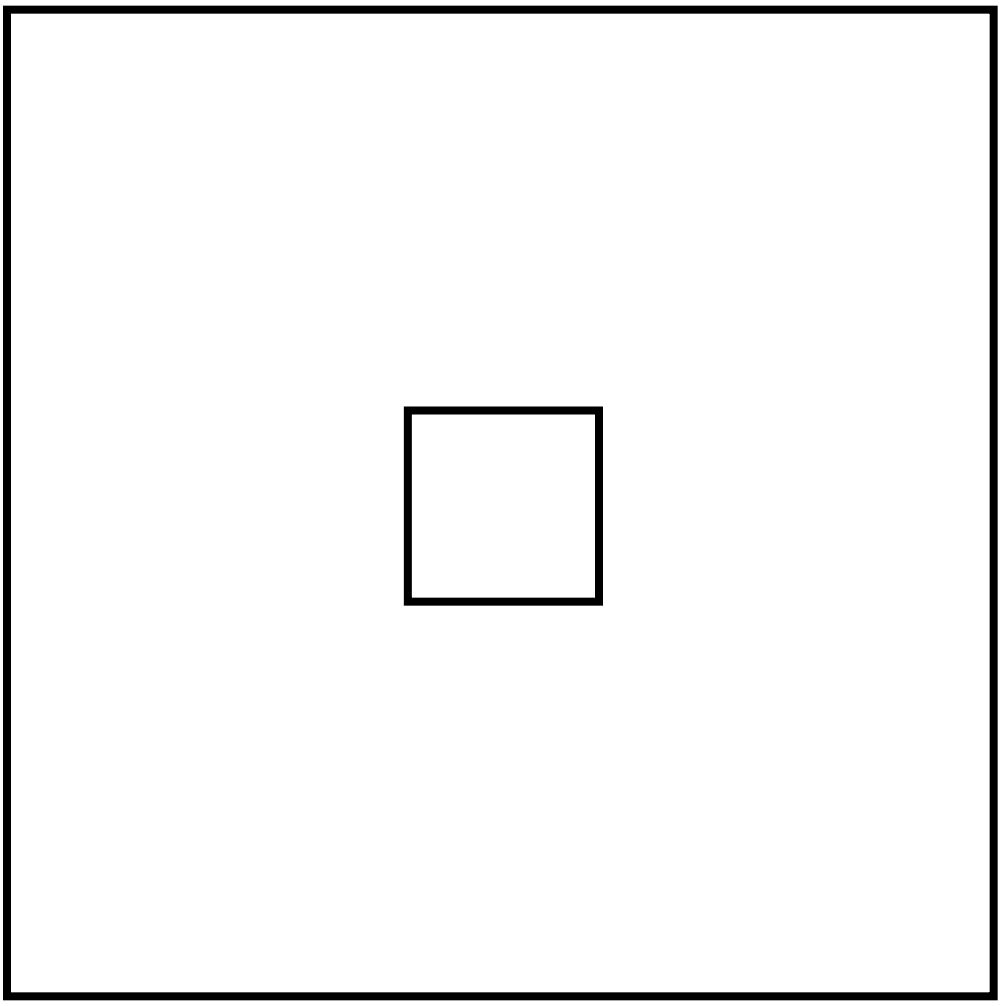




Get to The Center

You are on the bank of a square pond and can't swim. There is a square island in the middle of the pond. You have 2 planks to help you get onto the island but neither of them are long enough to reach across the water by them selves. You need to create a way to get to the island using only and no other materials.

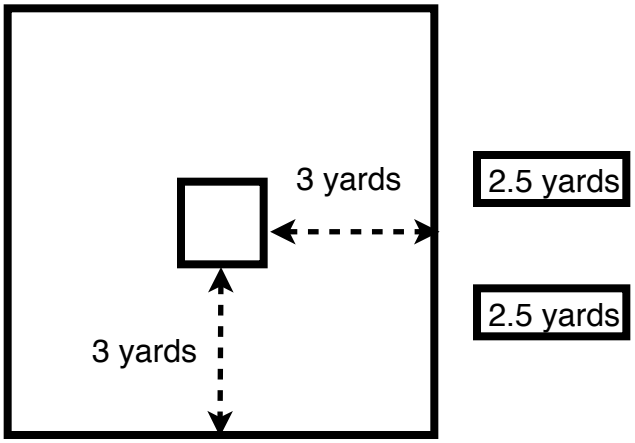
Cut out the two planks at the bottom of the page and place them so you can walk form the bank to the island



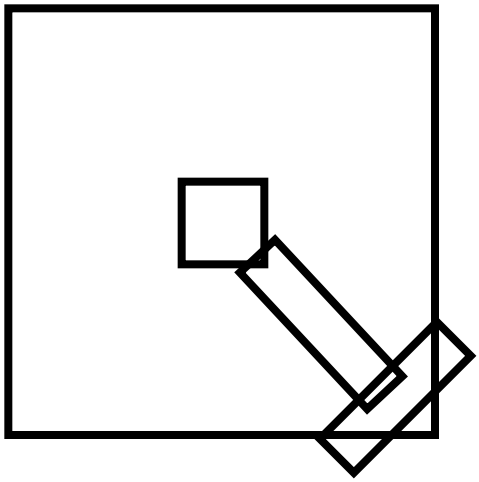
Plank

Plank

Problem



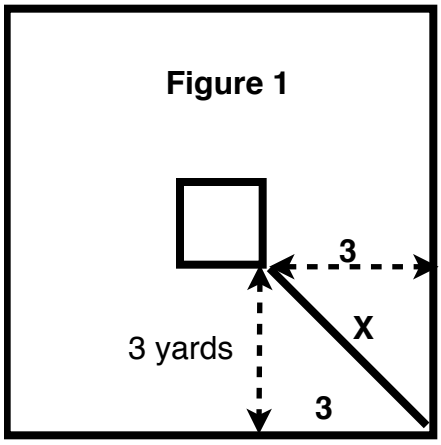
Posted Solution



I recently had a student show me this puzzle. They thought that the solution was very clever. The suggested solution is to place one board at the corner of the bank and then place the second board from the center of the first board to the corner of the island. It was impressed so I looked up other versions of this puzzle on the web. I found many problems that only varied from this problem by the length of the board and the width the waterway. No matter what lengths used, in every case the solution was the one shown above.

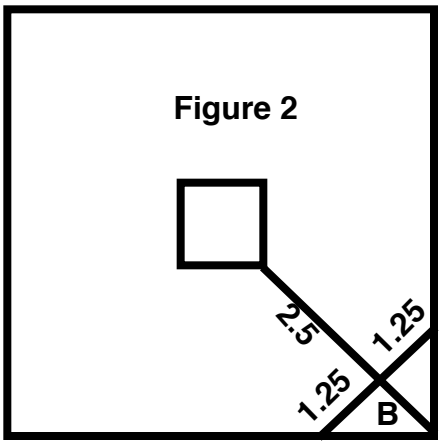
I decided to make sure that the solution shown was really possible. It is easy to draw a picture but another thing to have the numbers work out so when the picture is drawn to scale.

Figure 1



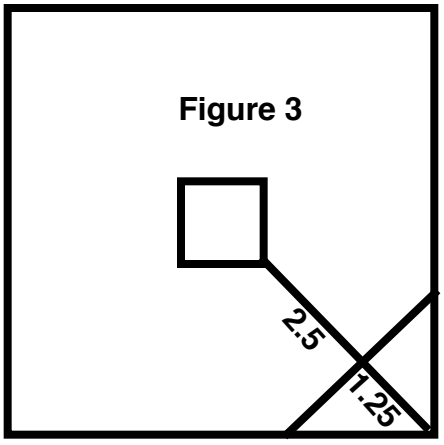
$$3^2 + 3^2 = x^2$$
$$9 + 9 = x^x$$
$$18 = x^2$$
$$x = 4.24 \text{ yards}$$

Figure 2



The length B is a leg of an isosceles triangle. The legs of the triangles are equal so $B = 1.25$ yards

Figure 3



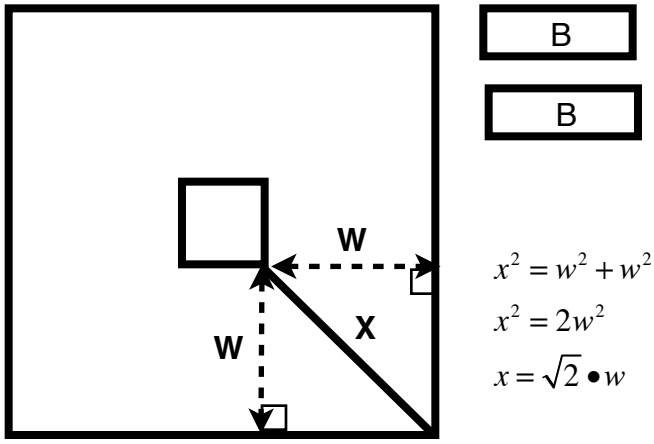
The longest distance that the 2 boards could cover is the length of the 2.5 yard plank added to the length of B in figure 2. B = 1.25 yards so the mamimum distance the 2 boards can cover is 3.75 yards. The actual distance from the corner of the bank to the coner of the island is 4.24 yards from figure 1. The two 2.5 yard long planks cannot cover this distance so the suggested solution does not work.

I found that the 3 yard wide waterway and the 2.5. yard boards does not work. I then checked the numbers that other versions of the puzzle used and I found that many of them did not work. I decided to find a generalized solution to the puzzle so I could tell how long the planks need to be for any given width of waterway.

Call the width of the water w and the distance from the corner of the bank to the corner of the island x. **Fig. 1**

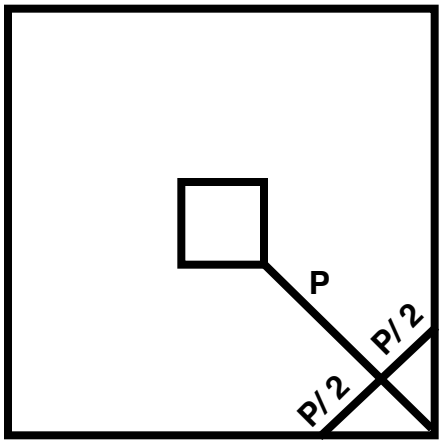
The 2 planks get placed to form a T at the corner of the bank. The plank that is placed at the corner will be bisected. **Fig. 2**

Figure 1



$$x^2 = w^2 + w^2$$
$$x^2 = 2w^2$$
$$x = \sqrt{2} \bullet w$$

Figure 2



The figures below are enlarger figures of the small right triangle at the corner of the bank shown in figure 2.

The 90 degree angle at the corner of the bank will be bisected. This forms two 45 degree angle. **Fig. 3**

The third angle of the small triangle must be 45 degree. The forms an isosceles right triangle with equal legs of p/ 2. **Fig. 4**

Figure 3

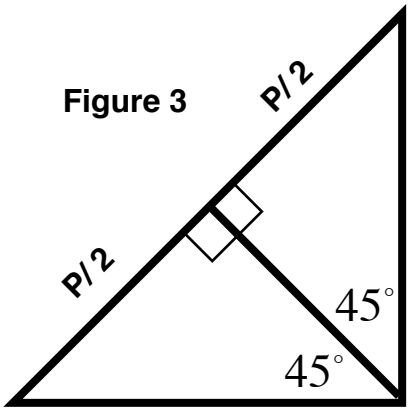
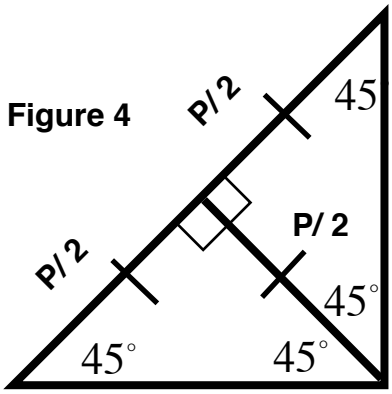


Figure 4



The total length the the 2 planks can cover is P + P/2 or 3p/2 from figure 4 and figure 5. Using Pythagorean Theorem we find that the length of the plank (p) must be at least $\frac{2\sqrt{2}}{3}$ times the width the waterway from figure 5..

Figure 5

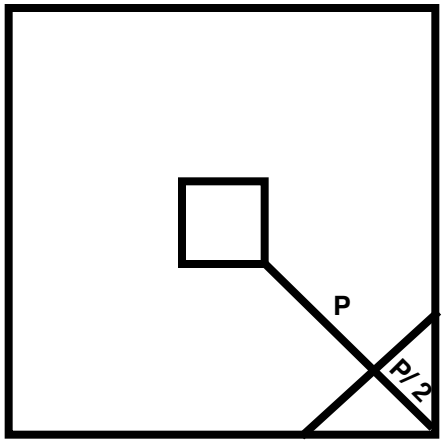
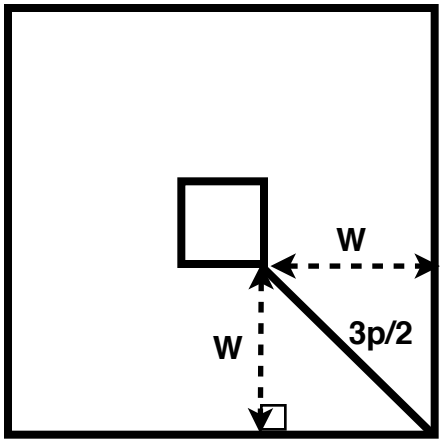


Figure 6



$$\left(\frac{3p}{2}\right)^2 = w^2 + w^2$$

$$\frac{9p^2}{4} = 2w^2$$

$$p^2 = \frac{8w^2}{9}$$

$$p = \frac{2\sqrt{2}}{3} \bullet w$$

Conclusion: Given any width the waterway W the length of the planks P must be longer than $\frac{2\sqrt{2}}{3} \bullet W$ or about $p > .95 \bullet W$

If the width of the waterway is 3 yards than the length of the plank must be $p > .95 \bullet 3$ so the plans must be at least 2.85 yards long. It is greater than 2.85 because the planks must overlap a bit where the intersect to form the t. The plank that rests on the corner of the island would need to overlap that corner a bit to be stable. This requires that the length of the plank p be longer than the stated 2.85 yards.

Critical Thinking Solutions

The puzzle was written so that the solution was to form a t with the planks at the corner of the bank. After students find it difficult to find a way to solve the problem with only the 2 planks they may start to suggest alternate solutions. Many suggestions for additional solutions have been suggested. These suggested solutions all use resources not mentioned by the problem. If the problem stated that the only materials you could use were the 2 planks then all the all alternate solutions would be eliminated. The second version of the puzzle that is listed on page 2 does just that. It states that you can only use the 2 planks and nothing else. If you allow any resources then the problem becomes trivial. You just get a helicopter and fly over or you get a bulldozer and create a path to walk over. Students will often reject these types of solutions as too far fetched. They may allow more reasonable solutions that suggest resources not listed but that could be reasonably expected to be available. It could be reasonable to claim the person was wearing a belt, had a shirt on, had a knife in their pocket or used a second person that came with them or walked by. The use of these reasonable extra resources can lead to creative solutions.

There is a value in examining some reasonable alternate solutions even if they require resources not mentioned in the problem. They show the results of creative thinking and can lead to good discussions between your students. I have listed a few of the ones I find the most fun to look at.

Lash then together solution.

The 2 planks have a total distance far in excess of the width of the waterway. If you overlapped the 2 planks and then lashed them together at the 2 points of intersection you would have a plank that would easily cross the width of the waterway. This would require 2 lashings. One could be a belt if you were wearing one. If not, make lashing out of both pants legs and walk across the planks in shorts.

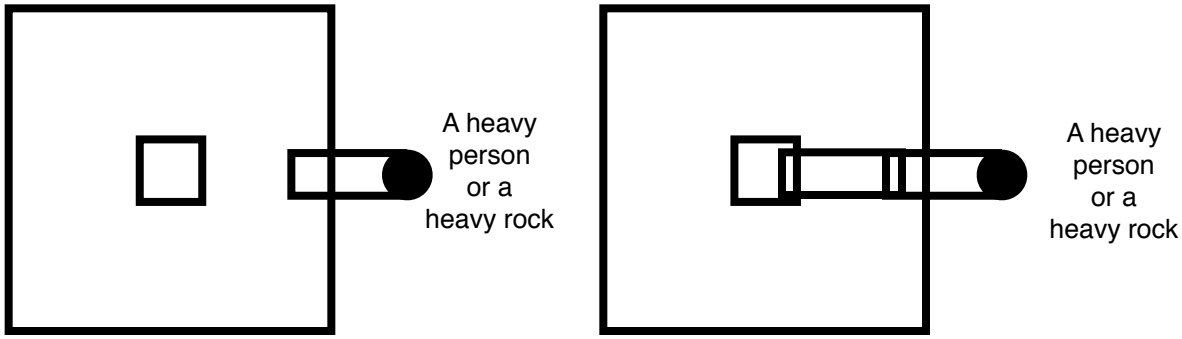
Make two stilts solution.

The problem does not state the depth of the water. It must be deep enough to stop you from just wading across. If it is not too deep you could make two stilts and use them to cross. This solution requires accepting that the person in the problem has a pocket knife. Use the knife to carve foot holds in each plank and use the stilts to walk across. This is a bit more far fetched than the last solution but it is creative and within reason.

The pole-vault solution

The length of the plank is always close to the width of the water way. Most problems give the length of the plank as several feet. This solution suggests you take one of the planks by the end and run towards the bank and set the other end in the dirt by the bank and pole vault over to the island. It does not allow you to get back. This is event more far fetched than the last solution but it is creative and may be within reason.

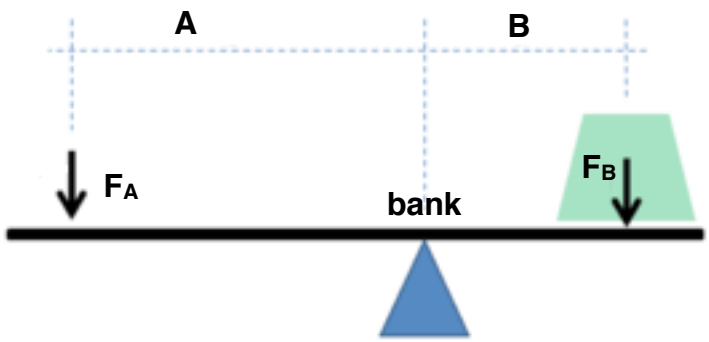
Cantilever 1 plank from the bank with a counterweight and lay the other one on it solution.



Place one plank so it's sticking out over the edge of the bank with the other end lying back from the bank. Have a heavy person stand on the end of the plank as shown or roll a heavy rock over the end of the plank. Walk out on this plank carrying the second plank. Lay this plank so it overlaps the first plank and reaches to the island.

Where does the second person come from? Either they came along with you or they walked by as you were trying to solve the problem. The other option is to have a heavy rook close by and roll it over the end of the plank. Perhaps this requirement makes this solution a bit more far fetched than the last solutions but it is creative.

I think that this is the most interesting of all the solutions. The reason is the first planks acts as a lever and the bank acts as the fulcrum point. This allows you to compute the mechanical advantage for the lever. The mechanical advantage or law of the lever is based on the weights of the people who are at the ends of the plank and the ratio of lengths of the plank on the shore (B) versus the length on the part of the plank over the water (A). It also assumes that the plank has the strength to bear the load of the person on the cantilevered end of the plank. This connects the solution of the problem to real science and brings it into the real world.



A (F_A) = A (F_B)

$$MA = \frac{F_A}{F_B} = \frac{A}{B}$$

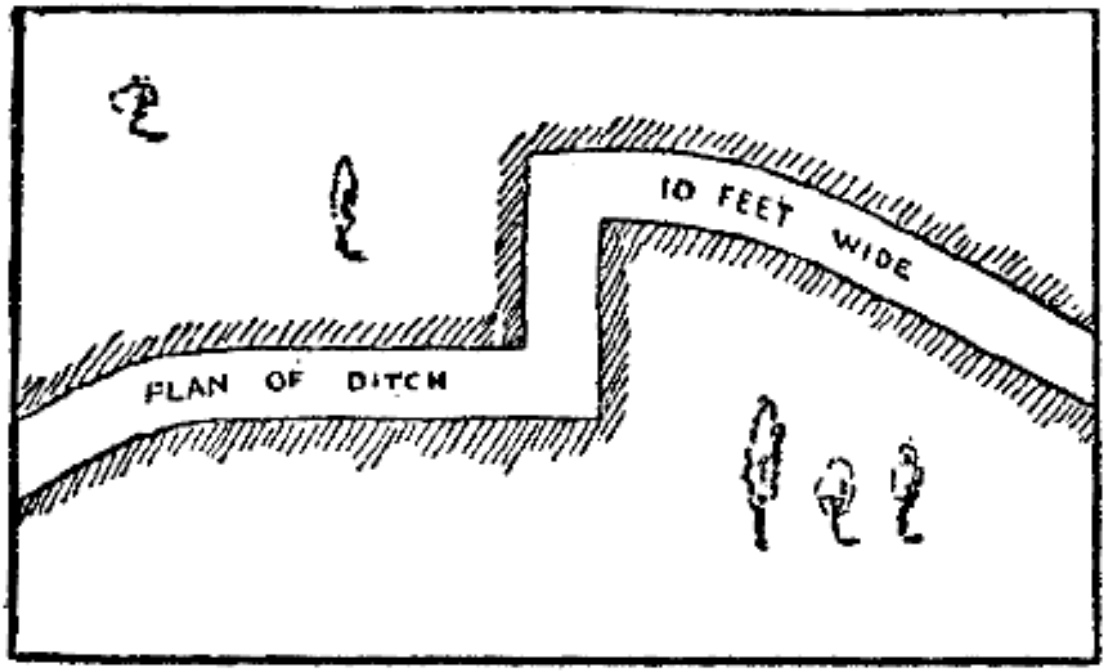
If the 2.5 yard board is placed so the 1.5 yards are on the bank and 1 yard is over the water than the mechanical advantage is $MA = \frac{1}{1.5} = .67 = \frac{F_A}{F_B}$ This means that $.67 \bullet F_B = F_A$. This says that to balance the lever the weight on the end of the plank on the bank needs to weigh on 2/3 of the weight of the person on the plank. If the person on the plank is 140 pounds and the plank they carry out with them weights 10 pounds for a total of 150 pounds than you would need a 100 pound weight on the other end.

The Origin Of The Puzzle

The origin of the problem we are discussing comes from one of the most famous mathematical puzzle books ever written, The Canterbury Puzzles and Other Curious Problems. It was written in 1907 by Henry Dudeney. He is one of the top 5 puzzle creators in the field of mathematical puzzles. The first part of the book features a series of puzzles based on the characters from The Canterbury Tales.

This puzzle asks you to cross the water using 8 planks.

54. Bridging the Ditch.

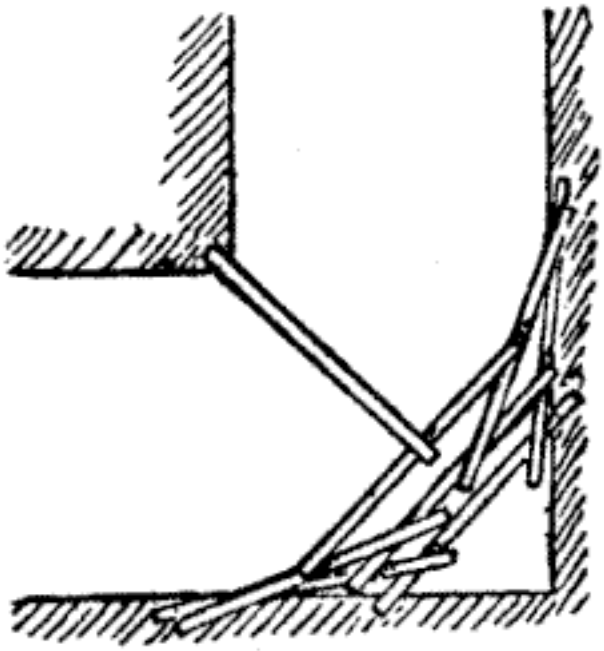


I now did truly think that at last was I a free man, but I had quite forgot that I must yet cross a deep ditch before I might get right away. **This ditch was 10 feet wide**, and I durst not attempt to jump it, as I had sprained an ankle in leaving the garden. Looking around for something to help me over my difficulty, I soon found **eight narrow planks** of wood lying together in a heap. **With these alone,** and **the planks were each no more than 9 feet long**, I did at last manage to make a bridge across the ditch. How was this done?

Ofttimes have I been asked by many that do know me to set forth to them the strange manner of my escape, which more than one hath deemed to be of a truth wonderful, albeit the feat was nothing astonishing withal if we do but remember that from my youth upwards I had trained my wit to the making and answering of cunning enigmas. And I do hold that **the study of such crafty matters is good,** not alone for the pleasure that is created thereby, but because a man may never be sure that in some sudden and untoward difficulty that may beset him in passing through this life of ours such strange learning may not serve his ends greatly, and, mayhap, help him out of many difficulties.

I am now an aged man, and have not quite lost all my taste for quaint puzzles and conceits. But, of a truth, never have I found greater pleasure in making out the answers to any of these things than I had in mastering them that did enable me, as the king's jester in disgrace, to gain my freedom from the castle dungeon and so save my life.

Solution:



The solution to this puzzle is best explained by the illustration. If he had placed his eight planks, in the manner shown, across the angle of the ditch, he would have been able to cross without much trouble. The king's jester might thus have well overcome all his difficulties and got safely away, as he has told us that he succeeded in doing.