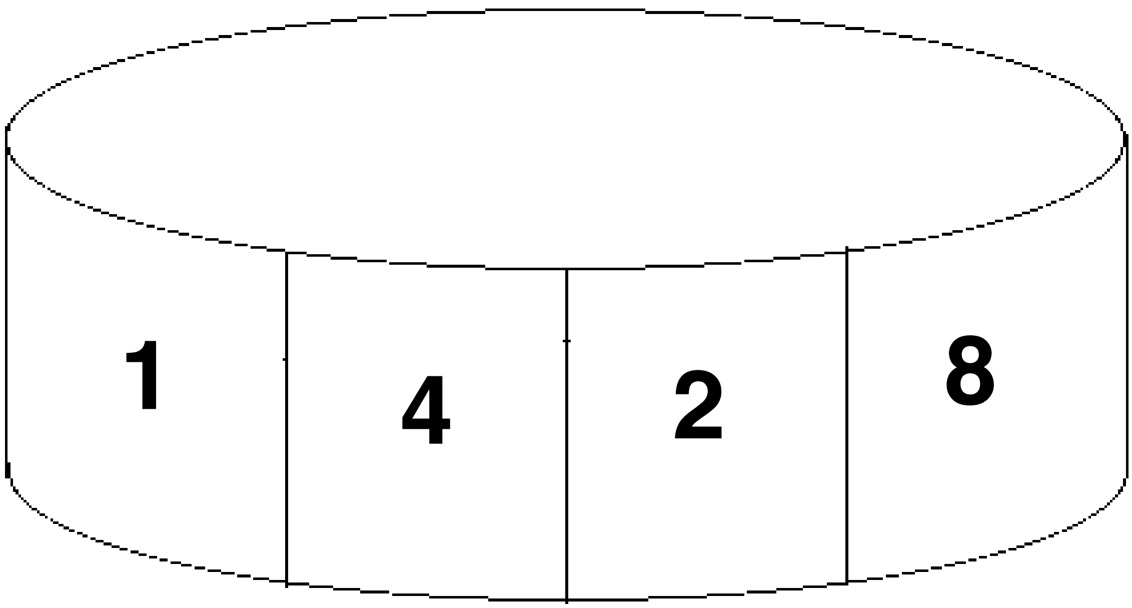
The x divided by 7 Chain Prediction

Ask a student to change 1/ 7 into a decimal by using long division. Tell them they need to keep dividing until the decimal starts to repeat. Division by 7 results in a repeating set of 6 numbers. They will need to continue the division at least seven times to start to see the repeating pattern for their division problem.

You take a paper ring of numbers and tear the ring so that the numbers showing are the repeated decimals in the correct order for the division problem they are doing.



You continue asking other students to convert 2/ 7 , 3/ 7 , 4/ 7 , 5/ 7 and 6/ 7 to a decimal and find the repeating decimal. Each time a student is given a fraction to convert you tear the chain to show the repeating digits in the correct order for their problem.

Example:

They **convert 2/ 7 into a decimal be long division.** As they start the problem **you tear the paper chain before the 2 so it reads**

2 8 5 7 1 4

When they do the division they get $\frac{2}{7} = 7\overline{)2.000000}$ with quotient $.\overline{285714}$ and see you were correct. By tearing the chain in the correct location you predict the answer to every division problem

Procedure:

Cut out the paper strips on the black line master page that has several strips like the one shown below.

1 4 2 8 5 7

Tape the ends of each strip together making a chain. Make the gap between the 1 and 7 have the same space between them as the other numbers.

Start by asking different students to convert 1/ 7, 2/ 7 , 3/ 7 , 4/ 7 , 5/ 7 or 6/ 7 to a decimal and find the repeating decimal. As you ask them to do the problem, tear the chain at the correct place and lay in on their desk with the numbers down. Repeat this for all the other students. When they get the answer have them check by turing over the prediction.

There are six division problems. They all have the same 6 digits in their answer but the order of the digits isn't the same.

$$\frac{1}{7} = \begin{array}{r} .\overline{142857} \\ 7\overline{)1.000000} \end{array} \qquad \frac{2}{7} = \begin{array}{r} .\overline{285714} \\ 7\overline{)2.000000} \end{array} \qquad \frac{3}{7} = \begin{array}{r} .\overline{425871} \\ 7\overline{)3.000000} \end{array}$$

$$\frac{4}{7} = \begin{array}{r} .\overline{571428} \\ 7\overline{)4.000000} \end{array} \qquad \frac{5}{7} = \begin{array}{r} .\overline{714285} \\ 7\overline{)5.000000} \end{array} \qquad \frac{6}{7} = \begin{array}{r} .\overline{857142} \\ 7\overline{)6.000000} \end{array}$$

Each division repeats after 6 divisions. The order of the repeating decimals is based on a chain of the numbers 142857

How do I know where to tear the chain ?

To find where the repeat begins start the division in your head.

1/ 7 says divide 7 into 1.000 **7 goes into 1.0 (ten) 1 times** with a remainder.

$\begin{array}{r} .1 \\ 7\overline{)1.0} \end{array}$ **The repeat starts at the digit 1. Tear BEFORE the 1.   1/ 7 = . 1 4 2 8 5 7**

2/ 7 says to divide 7 into 2.000   **7 goes into 2.0 (twenty) 2 times with a remainder.**

$\begin{array}{r} .2 \\ 7\overline{)2.0} \end{array}$ **The repeat starts at the digit 2. Tear BEFORE the 2.   2/ 7 =   . 2 8 5 7 1 4**

3/ 7 says to divide 7 into 3.000   **7 goes into 3.0 (thirty) 4 times with a remainder.**

$\begin{array}{r} .4 \\ 7\overline{)3.0} \end{array}$ **The repeat starts at the digit 4. Tear BEFORE the 4.   3/ 7 = . 4 2 8 5 7 1**

4/ 7 says to divide 7 into 4.000   **7 goes into 4.0 (forty) 2 times with a remainder.**

$\begin{array}{r} .5 \\ 7\overline{)4.0} \end{array}$ **The repeat starts at the digit 5. Tear BEFORE the 5.   4/ 7 =   . 5 7 1 4 2 8**

5/ 7 says to divide 7 into 5.000   **7 goes into 5.0 (fifty) 2 times with a remainder.**

$\begin{array}{r} .7 \\ 7\overline{)5.0} \end{array}$ **The repeat starts at the digit 7. Tear BEFORE the 7. 5/ 7 = . 7 1 4 2 8 5**

6/ 7 says to divide 7 into 6.000   **7 goes into 6.0 (sixty) 8 times with a remainder.**

$\begin{array}{r} .8 \\ 7\overline{)6.0} \end{array}$ **The repeat starts at the digit 8. Tear BEFORE the 8.   6/ 7 = . 8 5 7 1 4 2**

Cut out the strips of numbers and tape the ends of each strip together.

1	4	2	8	5	7
1	4	2	8	5	7
1	4	2	8	5	7
1	4	2	8	5	7
1	4	2	8	5	7

Teacher notes:

When we first start converting fractions to decimals we limit the fractions to the "easy ones." These are the basic fractions that terminate after 1 or 2 decimal places. 1/ 2 , 1/ 4 , 3/4 , 1/ 5, 2/ 5 , 3/ 5 , 4 / 5 , 1/10 3/ 10 , 7/ 10 and 9/ 10. When we get brave we use division by 8. division by 8 ends in 3 places so it is a good next step. That is the limit of the "good numbers." Division by 2 5 and 10 ends in the 10th place. Division by 4 ends in the 100th place and division by 8 ends in the 1000th place.

We then start to tackle the fractions with answers that have repeating digits. We start with division by 3 and 9 to get answers that repeat with a single digit. We use division by 6 and 11 to get answers that repeat is groups of 2 digits. we may try 1/ 12 to get numbers that repeat with 1 digit but the repeat does not start for 2 or 3 digits. That is a surprise for may students.

Have you noticed that we have used up all the numbers from 1 to 12 but have not used 7? This trick allows you to have them discover why division by 7 is always "left out".

It would be a great activity to try and create categories for each of the divisors from 2 to 20. They will discover that division by 7 and 14 are related. and why we never ask them to divide by 13, 17 and 19.

There are several ways to use this effect. You could do the trick for one student while the others watch. They will ask "how did you do that". Isn't it nice to have a student show interest in doing long division? Tell them you will help them see how it's done. List all 6 problems on the board and have each student do all 6 problems while you walk around and observe. Write the answers on the board and have them state the pattern. Show them the chain and let them see it has the pattern on digits on it. They will now ask how do you know where to tear the chain. Go over the procedure on page 2 with them.

I like having students write the problems on their own paper so I have less preparation to do. Some students benefit from seeing the problem set up. I write it on the board (write once, answer 35 students question). If you want to hand out a printed division problem they are listed below.

$$7\overline{)\,1.\ 0\ 0\ 0\ 0\ 0\ 0\ 0}$$

$$7\overline{)\,2.\ 0\ 0\ 0\ 0\ 0\ 0\ 0}$$

$$7\overline{)3.000000}$$

$$7\overline{)4.000000}$$

$$7\overline{)5.000000}$$

$$7\overline{)6.000000}$$

A follow up Trick with multiplication

Magic Products with a Magic Sum

The effect above is based on the fact that 1 divided by 7 is the repeating decimal .142857 and the fact that the fractions 2/ 7, 3/ 7 4/ 7 5/ 7 and 6/ 7 produce the same digits in the same cyclic order but start at different places on the cyclic chain

$$\frac{1}{7} = \begin{array}{r} .\overline{142857} \\ 7\overline{)1.00000} \end{array} \qquad \frac{2}{7} = \begin{array}{r} .\overline{285714} \\ 7\overline{)2.000000} \end{array} \qquad \frac{3}{7} = \begin{array}{r} .\overline{425871} \\ 7\overline{)3.000000} \end{array}$$

$$\frac{4}{7} = \begin{array}{r} .\overline{571428} \\ 7\overline{)4.000000} \end{array} \qquad \frac{5}{7} = \begin{array}{r} .\overline{714285} \\ 7\overline{)5.000000} \end{array} \qquad \frac{6}{7} = \begin{array}{r} .\overline{857142} \\ 7\overline{)6.000000} \end{array}$$

If you state the digits as the whole number 142, 857 you can create a new effect.

1. Hand out a piece of paper with the number 142,857 on it to 6 students. Have one students multiply this number by 1 and the next by 2 and the text by 3 and so on until the last one is asked to multiply that number by 6. Involve lost of students so they check each others answers.

2. While they are doing this make a 6 by 6 square grid with 36 squares.

3. Ask any students for the answer to their multiplication problem and pick any row on the square. Write their number on that row as they call it out. Be sure they have the correct answer. Continue this until all 6 rows are filled.

4. Remind them that they picked the order the row the numbers were put in so it really is a random placement of the 6 products. Display the square and ask then to sum the rows and columns of the square. The total will be 27.

Note: This is similar to a magic square but the diagonals will not total 27.

1 X 142857 = 1 4 2 8 5 7
2 X 142857 = 2 8 5 7 1 4
3 X 142857 = 4 2 8 5 7 1
4 X 142857 = 5 7 1 4 2 8
5 X 142857 = 7 1 4 2 8 5
6 X 142857 = 8 5 7 1 4 2

Notice that the digits listed vertically are the same as the digits listed horizontally

Magic Products with a Magic Sum

1 4 2, 8 5	1 4 2, 8 5 7
x 1	x 2
1 4 2, 8 5	1 4 2, 8 5 7
x 3	x 4
1 4 2, 8 5	1 4 2, 8 5 7
x 5	x 6