Coin Flipping Magic

Put 3 coins on the desk. They can be different denominations if you like. Have 2 or 3 students at a desk. It is always best to have a few students do a trick together, especially if you are going to turn your back on the process. The extra students help guarantee that the process is followed correctly and acts as a check on the answer or outcome. Turn around or move to the back of the class so you cannot see the coins.

Ask a student arrange the 3 coins in a horizontal line from left to right. Tell them that they may not switch the places the coins are in. The first coin must stay first and so on. They can have some coins heads up and some heads down in any order they want but they must have **at least one coin heads up and at least one coin tails up**. Have them tell you when they are finished.

Tell the students that they are going to be asked to flip over one coin in a position that you will select. Tell them that they may be asked to do this 1, 2 or perhaps 3 times with different coins at your direction.   Explain that as they touch the coins you will be able to feel the coins yourself and use that information to direct their flips in such a way that **the 3 coins will end up showing all heads or all tails**. Ask them if they are ready to start.

Step 1: Ask a student to **flip over the coin on the left end of the row.** Ask the student if the coins have all heads or all tails showing.   If so stop and state that they were able to arrange the coins any way they wanted and you were able to get the coins to match in 1 move. Stop here and take your bow.

Step 2. If step 1 did not cause all 3 coins to match then ask a student to **flip over the middle coin.** Ask the student if the coins have all heads or all tails showing. If so stop and state that they were able to arrange the coins any way they wanted but that you were able to detect the coins behavior and get the coins to match in 2 move. Stop here and take your bow.

Step 3. If the 3 coins do not match at this point say "I I must have made a mistake in visualizing your coins. Ah yes, I see. I made a mistake on the first step.   I shouldn't have flipped the coin on the left end in the first place. **Please flip the left coin on the left end over**." Now tell the students that you know that you have it right now and you are sure the coins are all the same. Stop here and take your bow.

Basic Instructions:

Step 1: Flip over the coin on the left side and ask if they all match. If not then**,**

Step 2. Flip over the middle coin and ask if they all match. If not then**,**

Step 2. Flip over the coin on the left side and state that the coins all match.

Example 1

Start

After Step 1

After Step 2

Done

Example 2

Start

After Step 1

After Step 2

After Step 3

Done

Students may find it surprising that just three flips are enough to equalize the three coins without ever knowing the state the coins started out in.

There are 6 possible states for the coins to start out in:

T H H   H T T   H H T   T T H   H T H   T H T

You can always cause one of 2 states, H H H or T T T , using no more than the three moves given.

Step 1 changes   T H H   and   H T T into all heads or all tails
Step 1and 2 changes   H H T   and   T T H into all heads or all tails
Step 1 , 2 and 3 changes   H T H and   T H T into all heads or all tails

History of the trick

This magic trick appears in Karl Fulves's book The Children's Magic Kit. According to that book, the trick was independently devised by Martin Gardner and Karl Fulves, based on a idea by Sam Schwartz. It works with coins or cards and can be extended to with dice with some work.

The moves used in the first example are not unique. You can do the trick a few times with the first set of moves and then change the rules or use a different set each time and perform the tick several times. Once you understand the basis of the moves it is easy to vary the rules each time. Otherwise write down each set and read them as you go.

Basic Instructions:

Step 1: Flip over the coin on the left side and ask if they all match. If not then**,**
Step 2. Flip over the middle coin and ask if they all match. If not then**,**
Step 2. Flip over the coin on the left side and state that the coins all match.

Variation 1:

Step 1: Flip over the coin on the right side and ask if they all match. If not then**,**
Step 2. Flip over the middle coin and ask if they all match. If not then**,**
Step 2. Flip over the coin on the right side and state that the coins all match.

Variation 2:

Step 1: Flip over the middle coin and ask if they all match. If not then**,**
Step 2. Flip over the coin on the right side and ask if they all match. If not then**,**
Step 2. Flip over the coin in the middle coin and state that the coins all match.

Variation 3:

Step 1: Flip over the middle coin and ask if they all match. If not then**,**
Step 2. Flip over the on the left side and ask if they all match. If not then**,**
Step 2. Flip over the coin on the middle coin and state that the coins all match.

Do you see the pattern?

Even Better: Have students develop a set of moves that work, test them on you. and then set them free to show the rest of the school.

The next process you may try is to flip more than one coin at a time, between asking for whether the coins are yet all the same. This flexibility cannot help equalize the coins any faster, but it can help obscure what the magician is doing.

You can flip 2 coins at once:

1. Flip the left and middle coins.
2. Flip the middle and right coins.
3. Flip the left and middle coins.

You can also vary the number of coins flipped:

1. Flip the left and middle coins.
2. Flip the left coin.
3. Flip the left and middle coins.

Students can be asked to "prove that each of the different sets of instructions ALWAYS work by testing them on the 6 possible states for the coins to start out in:

T H H   H T T   H H T   T T H   H T H   T H T

Are there a series of flips that will cause 4 coins , starting with at least 1 head and 1 tail to end up showing **all heads or all tails**?

Cause 4 coins with at least 1 head and 1 tail to have all heads or all tails in **at most 7 moves:**

1. Flip the first coin on the right over   and ask if they all match. If not then
2. Flip the second coin from the right over and ask if they all match. If not then
3. Flip the first coin on the right over   and ask if they all match. If not then
4. Flip the third coin from the right over   and ask if they all match. If not then
5. Flip the first coin on the right over   and ask if they all match. If not then
6. Flip the second coin from the right over and ask if they all match. If not then
7. Flip the first coin on the right over   and state that they have equal parity.

The maximum number of flips required to have al the coins match is
$$2^{n-1}-1$$ where n is the number of coins.

This family of tricks is best done with 3 coins, where the number of flips is small. With more that 3 coins the number of flips grows grows **exponentially**.

3 coins requires at most $2^{3-1}-1 = 2^2-1 = 3$ moves,

4 coins requires at most $2^{4-1}-1 = 2^3-1 = 7$ moves,

5 coins requires at most $2^{5-1}-1 = 2^4-1 = 15$ moves,

One solution to this exponential growth is to change the goal from the all-heads and all-tails problem to one where the number of heads and tails are equal:

Getting 3 heads and 3 tails with 6 coins in at most 4 moves:

This goal is exponentially easier to achieve. With at most n – 1 coin flips, you can force the number of heads and tails to be equal for n coins where n is an even number. The process is as follows: flip one coin at a time, in any order, until the goal has been reached.   The example below is for 6 coins. This algorithm will succeed before every coin has been flipped once. By randomly flipping all the coins in each move, you can expect to need approximately $\sqrt{n}$ moves. The practicality of this type of trick scales to larger n but is not that exciting as a magic trick.

1. Flip the second coin on the right over   and ask if they all match. If not then
2. Flip the fifth coin on the right over   and ask if they all match. If not then
3. Flip the third coin on the right over   and ask if they all match. If not then
4. Flip the first coin on the right over   and ask if they all match. If not then
5. Flip the fourth coin on the right over   and ask if they all match. If not then

A paper titled Coin Flipping Magic written by 4 MIT students is an excellent guide to the level of proof that can be developed around this topic. It was the source for this trick.   It is an excellent example of what trying to generalize a basic solution for 3 coins to a a solution for n coins entails. It really shows what a mathematician does and what a proofs are used fro. A web search will provide the link.