

Alphametic Puzzles

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1. Each letter or symbol represents only one single digit throughout the problem. If the letter A represents the number 5 then it must represent a 5 for every occurrence of the letter A
2. Each letter must be represented by a different digit.
3. When the letters are replaced by their digits, the resultant arithmetical operation must be correct.
4. The numbers cannot have a zero in the left most place value. For example, 020 is not allowed
5. The numerical base is base 10;

Example 2 is a classic, published in the July 1924 issue of Strand Magazine by Henry Dudeney.

Example 1

$$\begin{array}{r} \text{I} \\ + \text{B B} \\ \hline \text{I L L} \end{array}$$

If B=9 I=1 L=0

$$\begin{array}{r} 1 \\ + 99 \\ \hline 100 \end{array}$$

Which works

Example 2

$$\begin{array}{r} \text{SEND} \\ + \text{MORE} \\ \hline \text{MONEY} \end{array}$$

If D=7 E=5 M=1 N=6 O=0 R=8
S=9 Y=2 O=0 R=8 S=9 Y=2

$$\begin{array}{r} 9567 \\ + 1085 \\ \hline 10652 \end{array}$$

Which works

How to determine the solution for Example 1:

$$\begin{array}{r} I \\ + B B \\ \hline I L L \end{array}$$

The 2 digits in the ones place add to either L or $1 L$ where the one is carried to the 10s place because $I + L$ is more than 9 and less than 20. This must be the case in this problem because the 10 place in the problem has a 0 and a B that add to L . This means that $I + B > 9$

If there is a 1 carried over to the 10s place then $B + 1 = L$

B is at the left place so B cannot be 0. Neither can I

If $B + 1 = L$ and $I + B = L$ then $B + 1 = I + B$ and $I = 1$

The answer is a 3 digit number so $B + 1$ must add to > 9 so there is a one carried. This means that $I = 1$

$$\begin{array}{r} 1 \\ + B B \\ \hline 1 L L \end{array}$$

B must be 9 to allow the sum of $1 + B$ to carry a 1

$$\begin{array}{r} 1 \\ + 9 9 \\ \hline 1 L L \end{array}$$

$L = 0$

Solution

$$\begin{array}{r} 1 \\ + 9 9 \\ \hline 1 0 0 \end{array}$$

How to determine the solution for example 2

The 2 digits in the 1000's place have a 1 carried to the 1000's place so **M = 1**

$$\begin{array}{r} \text{SEND} \\ + \text{MORE} \\ \hline \text{MONEY} \end{array}$$

$$\begin{array}{r} \text{SEND} \\ + \text{1ORE} \\ \hline \text{1ONEY} \end{array}$$

In the 100's place $S + 1$ has a 1 carried over. $S + 1 > 9$ so $S > 8$ $S = 9$. $S + 1 = 10$ so $O = 0$

$$\begin{array}{r} \text{SEND} \\ + \text{1ORE} \\ \hline \text{1ONEY} \end{array}$$

$$\begin{array}{r} \text{SEND} \\ + \text{1ORE} \\ \hline \text{1ONEY} \end{array}$$

The 1000's column has $S + 1 = 10$ so $S = 9$

$$\begin{array}{r} \text{SEND} \\ + \text{1ORE} \\ \hline \text{1ONEY} \end{array}$$

$$\begin{array}{r} \text{9END} \\ + \text{1ORE} \\ \hline \text{1ONEY} \end{array}$$

In the 100's column $E + 0 = N$ and if there is no carry which leads to $E + N$ so there is a carry from the 10's column and $E + 1 = N$

If there were no carry in column 2, then $(N + R) \bmod 10 = E$, and $N = E + 1$, so $(E + 1 + R) \bmod 10 = E$ which means $(1 + R) \bmod 10 = 0$, so $R = 9$. But $S = 9$, so there must be a carry in column 2 so **R = 8**.

$$\begin{array}{r} \text{9END} \\ + \text{1ORE} \\ \hline \text{1ONEY} \end{array}$$

$$\begin{array}{r} \text{9END} \\ + \text{108E} \\ \hline \text{1ONEY} \end{array}$$

To produce a carry in column 2, we must have $D + E = 10 + Y$.

Y is at least 2 so $D + E$ is at least 12.

The only two pairs of available numbers that sum to at least 12 are (5,7) and (6,7) so either $E = 7$ or $D = 7$. Since $N = E + 1$, E can't be 7 because then $N = 8 = R$ so **D = 7**.

$$\begin{array}{r} \text{9END} \\ + \text{108E} \\ \hline \text{1ONEY} \end{array}$$

$$\begin{array}{r} \text{9EN7} \\ + \text{108E} \\ \hline \text{1ONEY} \end{array}$$

We have $N = E + 1$ and E can't be 6 because then $N = 7$ which is used so **E = 5** and **N = 6**. $D + E = 12$ so **Y = 2**.

$$\begin{array}{r} 9567 \\ + 1085 \\ \hline 10652 \end{array}$$

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Problem 1

$$\begin{array}{r} \text{HERE} \\ + \text{SHE} \\ \hline \text{COMES} \end{array}$$

Problem 2

$$\begin{array}{r} \text{NUMBER} \\ + \text{NUMBER} \\ \hline \text{PUZZLE} \end{array}$$

Problem 3

$$\begin{array}{r} \text{EAT} \\ + \text{THAT} \\ \hline \text{APPLE} \end{array}$$

Problem 4

$$\begin{array}{r} \text{HES} \\ + \text{THE} \\ \hline \text{BEST} \end{array}$$

Problem 5

$$\begin{array}{r} \text{OOOH} \\ + \text{FOOD} \\ \hline \text{FIGHT} \end{array}$$

Problem 6

$$\begin{array}{r} \text{FIFTY} \\ + \text{STATES} \\ \hline \text{AMERICA} \end{array}$$

Problem 7

CIRCLE
CIRCLE
+ CIRCLE
SPHERE

Problem 8

STORE
AND
+ NAME
BRANDS

Problem 9

GREEN
+ ORANGE
COLORS

Problem 10

CROSS
+ ROADS
DANGER

Problem 11

ELF
+ ELF
FOOD

Problem 12

SQUARE
+ DANCE
DANCER

Problem 13

MARTIN
+ GARDNER
RETIRES

Problem 14

HEAD
+ TOE
REACH

Problem 15

ADAM
AND
+ EVE
MOVED

Problem 16

MOON
MEN
+ CAN
REACH

Solutions

Problem 1 Solution

```
  HERE
+  SHE
-----
  COMES    has 1 solution.
```

```
 9454    C=1 E=4 H=9 M=3 O=0 R=5 S=8
+ 894
-----
10348
```

Problem 2 Solution

```
  NUMBER
+ NUMBER
- -----
  PUZZLE    has 1 solution.
```

```
 201689    B=6 E=8 L=7 M=1 N=2 P=4 R=9 U=0 Z=3
+201689
-----
 403378
```

Problem 3 Solution

```
  EAT
+ THAT
-----
  APPLE    has 1 solution.
```

```
  819    A=1 E=8 H=2 L=3 P=0 T=9
+9219
-----
10038
```

Problem 4 Solution

```
  HES
+ THE
-----
  BEST    has 1 solution.
```

```
  426    B=1 E=2 H=4 S=6 T=8
+842
-----
1268
```

Problem 5 Solution

```
OOOH
+FOOD
-----
FIGHT has 1 solution.
```

```
8886      D=3 F=1 G=7 H=6 I=0 O=8 T=9
+1883
-----
10769
```

Problem 6 Solution

```
FIFTY
+STATES
-----
AMERICA has 1 solution.
```

```
65682      A=1 C=3 E=4 F=6 I=5 M=0 R=7 S=9 T=8 Y=2
+981849
-----
1047531
```

Problem 7 Solution

```
CIRCLE
CIRCLE
+CIRCLE
-----
SPHERE has 1 solution.
```

```
108195      C=1 E=5 H=4 I=0 L=9 P=2 R=8 S=3
108195
+108195
-----
324585
```

Problem 8 Solution

```
STORE
AND
+ NAME
-----
BRANDS has 1 solution.
```

```
94307      A=2 B=1 D=5 E=7 M=6 N=8 O=3 R=0 S=9 T=4
285
+ 8267
-----
102859
```

Problem 9 Solution

GREEN
+ORANGE

COLORS has 1 solution.

83446 A=5 C=2 E=4 G=8 L=9 N=6 O=1 R=3 S=0

+135684

219130

Problem 10 Solution

CROSS
+ROADS

DANGER has 1 solution.

96233 A=5 C=9 D=1 E=4 G=7 N=8 O=2 R=6 S=3

+62513

158746

Problem 11 Solution

ELF
+ELF

FOOL has 1 solution.

721 E=7 F=1 L=2 O=4

+721

1442

Problem 12 Solution

SQUARE
+ DANCE

DANCER has 1 solution.

824163 A=1 C=7 D=9 E=3 N=5 Q=2 R=6 S=8 U=4

+ 91573

915736

Problem 13 Solution

MARTIN
+GARDNER

RETIRES has 1 solution.
It is:

862903 A=6 D=7 E=4 G=1 I=0 M=8 N=3 R=2 S=5 T=9
+1627342

2490245

Problem 14 Solution

HEAD
+ TOE

WAIST has 1 solution.

9708 A=0 D=8 E=7 H=9 I=2 O=3 S=4 T=5 W=1
+ 537

10245

Problem 15 Solution

ADAM
AND
+ EVE

MOVED has 1 solution.

8581 A=8 D=5 E=9 M=1 N=7 O=0 V=3
875
+ 939

10395

Problem 16 Solution

MOON
MEN
+ CAN

REACH has 1 solution.

9552 A=8 C=3 E=0 H=6 M=9 N=2 O=5 R=1
902
+ 382

10836

History

Cryptarithms

Cryptarithms are a type of mathematical puzzle in which the digits are replaced by symbols (typically letters of the alphabet). For example:

$$\sqrt{\text{CAREER}} = \text{RUT}$$

The earliest known cryptarithms were found in China. So the Chinese have been given the credit for devising this kind of puzzle. In India during the Middle Ages, a genre of cryptarithms nowadays known as "mathematical restorations" appeared.

Alphametic puzzles are a subset of cryptarithms. The inventor of alphametic puzzles is not known. The journal Sphinx was published in Belgium in the French language and was entirely devoted to recreational mathematics. The great work of boosting and popularizing modern cryptarithmic puzzles was done by the editors and readers of Sphinx from 1931 to 1939



It was in the Sphinx edition of May 1931 that M. Vatriquant first introduced these type of puzzles by proposing the following puzzle.

$$\begin{array}{r} ABC \\ + DE \\ \hline HGBC \end{array}$$

A. H. Hunter introduced the word "alphametic" to designate cryptarithms whose letters form meaningful words or phrases. Alphametic puzzles were submitted by the subscribers to the magazine. To boost interest editors used to organize puzzle contests paying cash prizes to the winners.

The attraction to alphametics may be that they were hard to construct. First of all only 10 different letters of the alphabet (at most) can be used. This, naturally, makes it hard to write phrases or sentences that read well. Even if we write down a nice phrase or sentence representing a prospective alphametic, the odds that the alphametic will actually be solvable are pretty small. Finally, you add the additional constraint that there be one solution. you can see that finding a clever word phrase that has 1 solution is hard. Some people drop the unique solution so they can produce some clever phrases.

The difficulty of finding solutions or unique) solution was greatly reduced by the develop of computer algorithms. The web pages at trumancollins.net provide a program where you put in your phrase and it provides all the possible solutions. This provides A rapid way to test you phrase and get a solution if one exists. It is still hard to find clever ones, even with the program.

Cryptarithmic Puzzle Solver

At Kobe University, Japan, a computer running LLP (Linear Logic Programming Language) solves addition cryptarithms on-line. You enter the problem, press the "Solve" button, and after "n" microseconds there comes the solution! Never fails.

<http://bach.istc.kobe-u.ac.jp/llp/crypt.html>

Alphametic Puzzle Solver

The following computer on line solver works very well.

http://www.trumancollins.net/truman/alphamet/alpha_solve.shtml

Other Problems

```
SO
+SO
---
TOO has 1 solution.
```

```
50    O=0 S=5 T=1
+50
---
100
```

MACHU + PICCHU = INDIAN

```
 MACHU
+PICCHU
-----
INDIAN
has 2 solutions in base 10.
```

They are:

```
 81706    A=1 C=7 D=9 H=0 I=4 M=8 N=2 P=3 U=6
+347706
-----
429412
```

```
 13465    A=3 C=4 D=7 H=6 I=9 M=1 N=0 P=8 U=5
+894465
-----
907930
```

NO + GUN + NO = HUNT

```
NO
GUN
+ NO
----
HUNT has 1 solution.
```

```
87    G=9 H=1 N=8 O=7 T=2 U=0
908
+ 87
----
1082
```

US + AS = ALL

US
+AS

ALL
has 1 solution.

85 A=1 L=0 S=5 U=8
+15

100

AGONY + JOY = GUILT

AGONY
+ JOY

GUILT has 1 solution.

89562 A=8 G=9 I=3 J=7 L=1 N=6 O=5 T=4 U=0 Y=2
+ 752

90314

UK = USA = USSR = ABOMB

UK
USA
+USSR

ABOMB has 4 solutions. One is below:

95 A=1 B=0 K=5 M=2 O=7 R=4 S=6 U=9
961
+9664

10720

Problem.

TEACH
STATS
+ WELL

EITEL has 6 solutions. One is shown below.

28756 A=7 C=5 E=8 H=6 I=1 L=0 S=4 T=2 W=9
42724
+ 9800

81280

Problem.

STORE
AND
+ NAME

BRANDS has 1 solution.

94307 A=2 B=1 D=5 E=7 M=6 N=8 O=3 R=0 S=9 T=4
 285
+ 8267

102859

Problem.

LEARN
STATS
+ WELL

EITEL has 9 solutions. One is shown below.

24067 A=0 E=4 H=3 I=1 L=2 M=9 N=7 R=6 T=5 W=8
 9053
+ 8422

41542

Problem.

TEACH
MATH
+ WELL

EITEL has 76 solutions. One is shown below.

12960 A=9 C=6 E=2 H=0 I=4 L=5 M=7 T=1 W=3
 7910
+ 3255

24125

Problem

PEAR
+APPLE

GRAPE has 1 solution.

5280 A=8 E=2 G=9 L=7 P=5 R=0
+85572

90852

Problem

TED
HAS
+GOOD

TASTE has 1 solution.

134 A=0 D=4 E=3 G=9 H=6 O=7 S=5 T=1
605
+9774

10513

Problem

FATHER
+MOTHER

PARENT
has 2 solutions in base 10.

They are:

286753 A=8 E=5 F=2 H=7 M=1 N=0 O=9 P=4 R=3 T=6
+196753

483506

186753 A=8 E=5 F=1 H=7 M=2 N=0 O=9 P=4 R=3 T=6
+296753

483506

Problem.

LEARN
MATH
+ WELL

EITEL has 4 solutions. One is shown below

24067 A=0 E=4 H=3 I=1 L=2 M=9 N=7 R=6 T=5 W=8
9053
+ 8422

41542

Problem

THREE
THREE
TWO
TWO
+ ONE

ELEVEN has 1 solution.

84611 E=1 H=4 L=7 N=9 O=3 R=6 T=8 V=2 W=0
84611
803
803
+ 391

171219

Problem

SEVEN
SEVEN
+ SIX

TWENTY has 1 solution.

68782 E=8 I=5 N=2 S=6 T=1 V=7 W=3 X=0 Y=4
68782
+ 650

138214

Alphametic Puzzle Solver

The following computer on line solver works very well.

http://www.trumancollins.net/truman/alphamet/alpha_solve.shtml

Use the solver above to find the solutions to the following Alphametric puzzles.

COPY + PASTE + SAVE = TOOLS

THIS + IS + A + GREAT + TIME = WASTER

HERE + THEY + GO = AGAIN

AMELIA + PEELED + A = BANANA.

THIS + IS + A + GREAT + TIME = WASTER

XMAS + MAIL + EARLY + PLEASE

LETTERS + ALPHABET = SCRABBLE

WHEN + IN + ROME + BE + A = ROMAN

GEE + I + SEE + A + RARE + MAGIC + SQUARE

TERRIBLE + NUMBER = THIRTEEN

EARTH + AIR + FIRE + WATER = NATURE

SATURN + URANUS + NEPTUNE + PLUTO = PLANETS

GEORGIA + OREGON + VERMONT = VIRGINIA

TOO + TOO + TOO + TOO = GOOD

OLD + OLD + OLD = GOOD

TED + HAS + GOOD = TASTE

NOTICE + NICE = PRICES

BARREL + BROOMS + SHOVELS

EIGHT + EIGHT + TWO + ONE + ONE = TWENTY

ELEVEN + NINE + FIVE + FIVE = THIRTY

NINE + SEVEN + SEVEN + SEVEN = THIRTY

TEXAS + NEVADA = ALASKA

KANSAS + OREGON = ARIZONA

EARTH + URANUS = SATURN

PEAR + APPLE = GRAPE

DATE + BANANA = CHERRY

NEON + NICKEL = COBALT

LEAD + SILVER = RADIUM