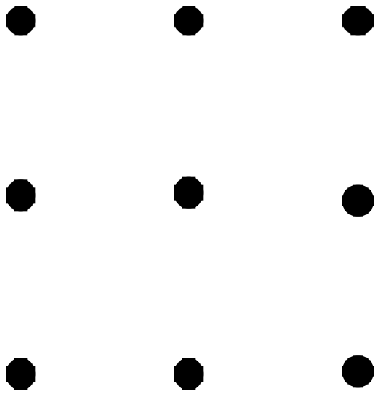
The Nine Dot Puzzle

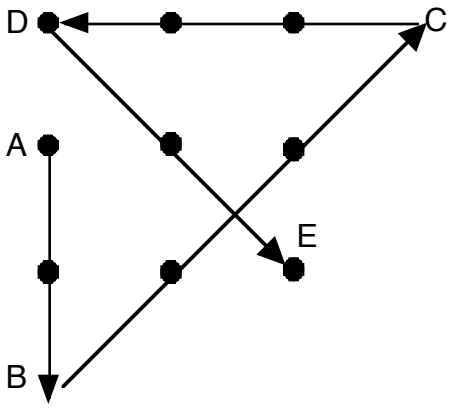
Use a pencil to draw four continuous straight line segments which go through the middle of all 9 dots without taking the pencil off the paper.



Solution

Start at A go to B then C
then D then E

You need to draw line
segments that go
beyond the dots
to solve the problem



The 9 dot puzzle is an old puzzle. It appears in Sam Loyd's 1914 puzzle book. It is a very well known problem used by many psychologists to explain the mechanism of 'unblocking' the mind in problem solving activities. It is probable that this brainteaser gave origin to the expression 'thinking outside the box'.

One difficulty people have in solving the puzzle is the tendency to make the incorrect assumption that the line segments must stay within the perimeter of the 9 dots. Many people also make the incorrect assumption that each line segment must start and end on a dot. These initial false assumptions cause the person to limit themselves to a point the solution is not possible No matter how many times they try to draw four straight line segments without lifting the pencil there is always a dot remaining that was not crossed. You will often see the person trying the same pathway many times. Once they drop that limiting thought of a boundary (or are told they can do so) the solution seems to be found very fast.

A few free thinkers may try to use curved “line segments” to get a solution. That leads to a good discussion about setting initial definitions. Some would say the definition of a line segment means that it is straight. Others may argue that if you mean straight you should say so.

Before you start to solve a problem you should examine carefully any assumptions that you may be imposing on yourself that are not actually stated in the problem. The best way to do that is to use clarifying questions.

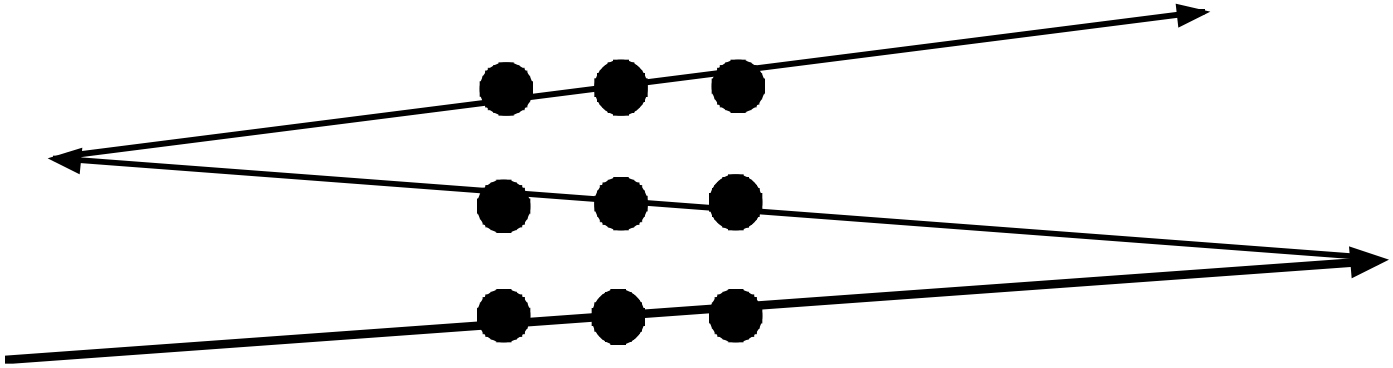
Can the line segments cross?
Do I need to start at a dot?
Can a dot be crossed more that once.
Can a dot be crossed or touched more that once
Must the line segments be straight?
Is there a limit as to how long each line segment can be?

Theory Versus the Real World

In Euclidean geometry we do not define a point as having a dimensional attribute. Points do not have an area, volume, width or height. A common interpretation is that the concept of a point is meant to capture the notion of an object, with no properties, in a unique **location** in Euclidean space.

But this is a 9 DOT puzzle not a 9 POINT puzzle.

Dots do have a dimensional attribute. The fact that the dot has some height leads to this solution that solves the puzzle using 3 line segments.



No matter how small the height of your dots 3 lines can be drawn to go through all 9 dots. The smaller the dots the longer the lines will need to be for this solution to work.

A good student may say that even if we allow the dot to have a height the solution above does not solve the puzzle you presented. You asked for 4 line segments and they used 3. The solution above world work if you had asked for 3 line segments. Once again clarity at the start of a problem is important

We can eliminate the discussion about the 3 line solution by changing the wording of the problem. If we change the wording of the problem to "**go through the middle of all 9 dots"** we eliminate the 3 line segment solution. The top line must have some slope for the above solution to work and that line cannot go through the middle of all the top 3 points in the first row. This would force the 3 lines thought the 3 rows of dots to be parallel and ruin this as a solution.

The wording below is the one I use when I do the puzzle.

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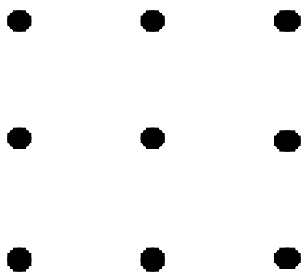
Draw with a pencil four continuous straight line segments which go through t**he middle** of all 9 dots without taking the pencil off the paper.



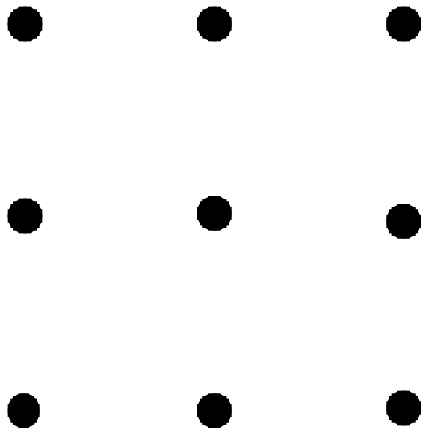
What about a 3 dimensional solution?

The problem was printed on on a paper or viewed on a flat screen so the assumption is that the surface the dots lie on must be flat. What if the dots were on a rubber sheet and that sheet was stretched and placed on a classroom globe. How would that change the problem? You could draw a single line that wraps around the globe 3 times and goes through the 9 points. Would that "line" really meet the definition of a line. Not in Euclidean geometry. Euclidean geometry, all work is done on a flat surface. That is why we call that geometry Plane Euclidean Geometry. Spherical geometry would allow this solution.

We could change the wording of the problem to state the puzzle must be done on the flat surface of the paper. That would eliminate the 3 dimensional solution.

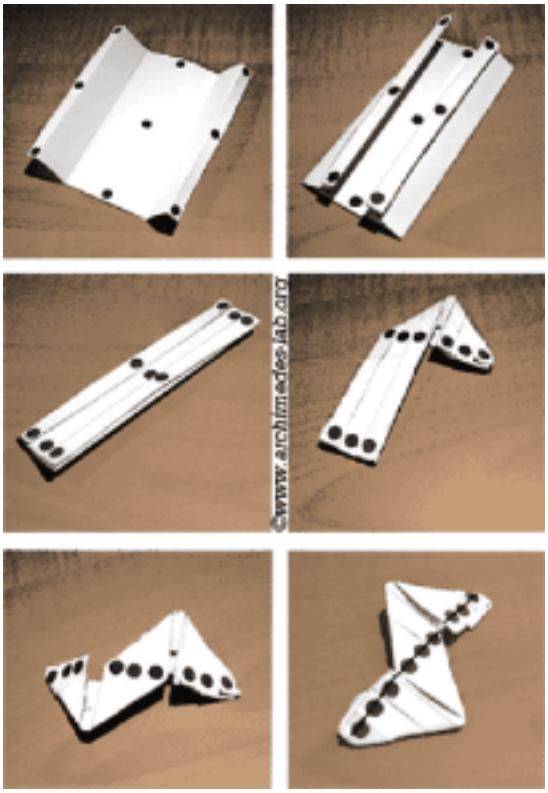
The Nine Dot Puzzle

Use a pencil to draw four continuous straight line segments which go through t**he middle** of all 9 dots without taking the pencil off the **flat surface** of the paper.



The " I redefine what a continuos line and flat surface mean" solution

What do you mean by a flat surface or a continuous line" If I fold the paper a few times as shown in the pictures below you do get the 9 dots in a row A straight line will go through the middle of the 9 dots. As the pencil moves along the paper it passes over "gaps" where the folds are. You may not consider this a continuous line as it bridges several gas in the paper. Clearly if you open up the paper the line is not continuous. Depending on how you define continuos this may or may not be a good solution. The 9 dots started out on a flat surface. After the folding process the dots are all in line but is the surface that are on flat? It depends on what you mean by a flat surface. Either way it is another impressive way to try and solve the problem by challenging the basic assumptions we started the problem with.



*Source: **MateMagica**, Sarcone & Waeber, ISBN: 88-89197-56-0.*

We could change the wording of the problem to state the puzzle must be done on the flat paper surface that cannot be folded in any way. That would eliminate this clever solution.

The Nine Dot Puzzle

Use a pencil to draw four continuous **straight** line segments which go through t**he middle** of all 9 dots without taking the pencil off the **flat surface** of the paper. **You cannot fold the paper** in any way.

This puzzle with its many possible solutions is rich in mathematical concepts and vocabulary. You could use the puzzle to help your students see why we try to be very specific in the wording of our definitions and theorems. Students always want a shortcut way to write out their work. This may help them see why you require such precise wording. Unfortunately that precise wording is almost always longer then the quick and easy one they want to use.

The Nine Dot Puzzle

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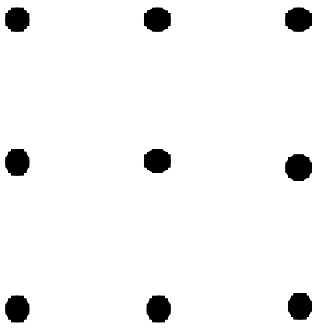

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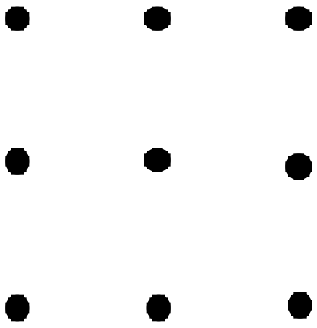
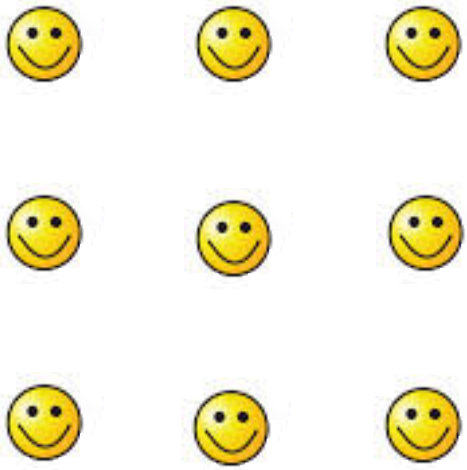


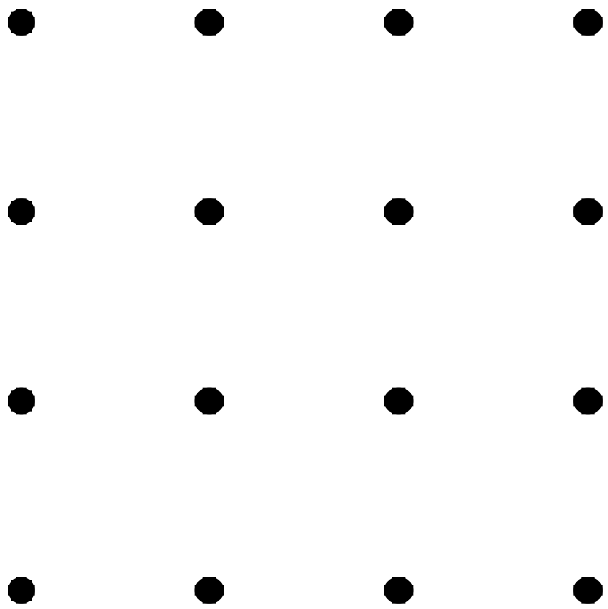
The Nine Smiley Faces Puzzle

Use a pencil to draw four continuous straight line segments which go through the middle of all 9 smiley faces without taking the pencil off the flat surface of the paper.

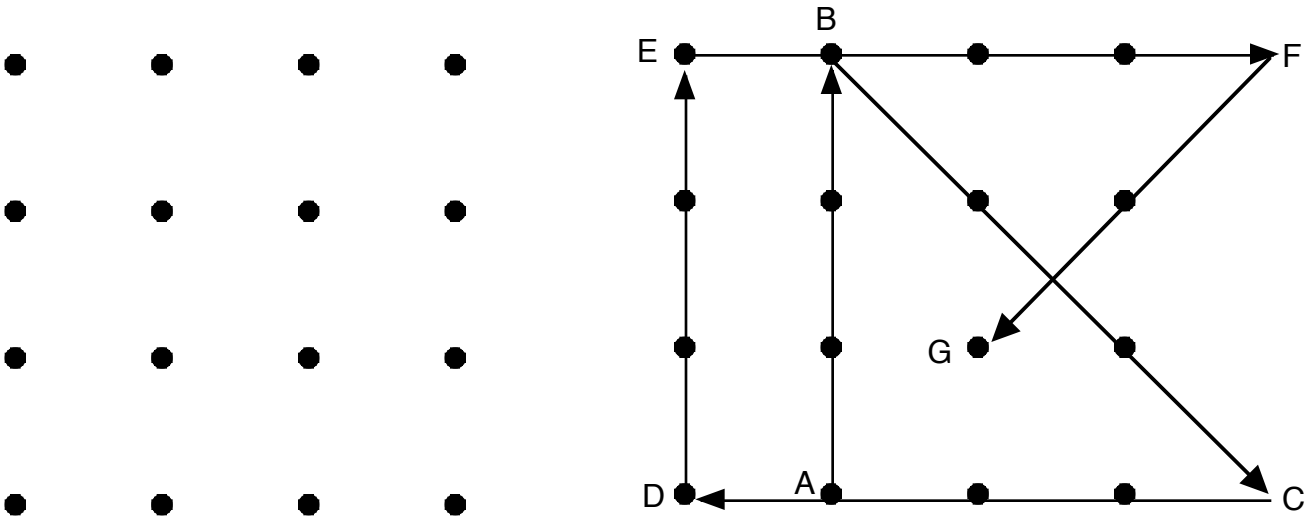


The 16 Dot Puzzle

Use a pencil to draw 6 continuous straight line segments which go through the middle of all 16 dots without taking the pencil off the paper.



A possible solution.



Look at the assumptions that we need to overcome by asking clarifying questions.

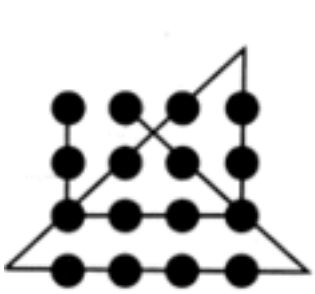
1. Can the segments cross over each other? Yes

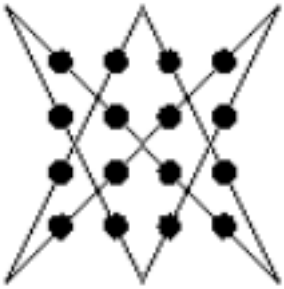
2. Can 2 segments go through or touch the same point? Yes and Yes.

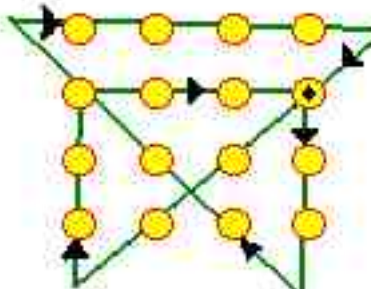
There are several solutions to the puzzle. It would be a great class project to find as many different solutions as you can. Even that task has its complications. Some solutions are just rotations or reflections of the solution shown above. These should not count. Excluding rotations or reflections there are more then 10 different solutions.

Is the internet killing problem solving?

In some ways it is indeed killing the thrill of discovery. A fast web search will find several solutions to the 16 dot puzzle shown without the student doing any work.







A very common site on the internet has students e-mails in a “I need some help” question. The response is the answer. One site that provides answers uses the name **Cheatwell.** I guess that's their idea of help. Just give me the answer so I can hand it in as proof I have a solution. The reason for the use of puzzles is to try and teach certain skills: try an easier version of the problem, rethink basic assumptions, look at simpler problems and many other strategies that we want students to learn and use. All this can be lost by just looking up the answer.

For this reason I often try to use a puzzle in class and have them complete the work before they leave. This may not be the perfect solution but it does help them focus on their work and not just finding an answer.