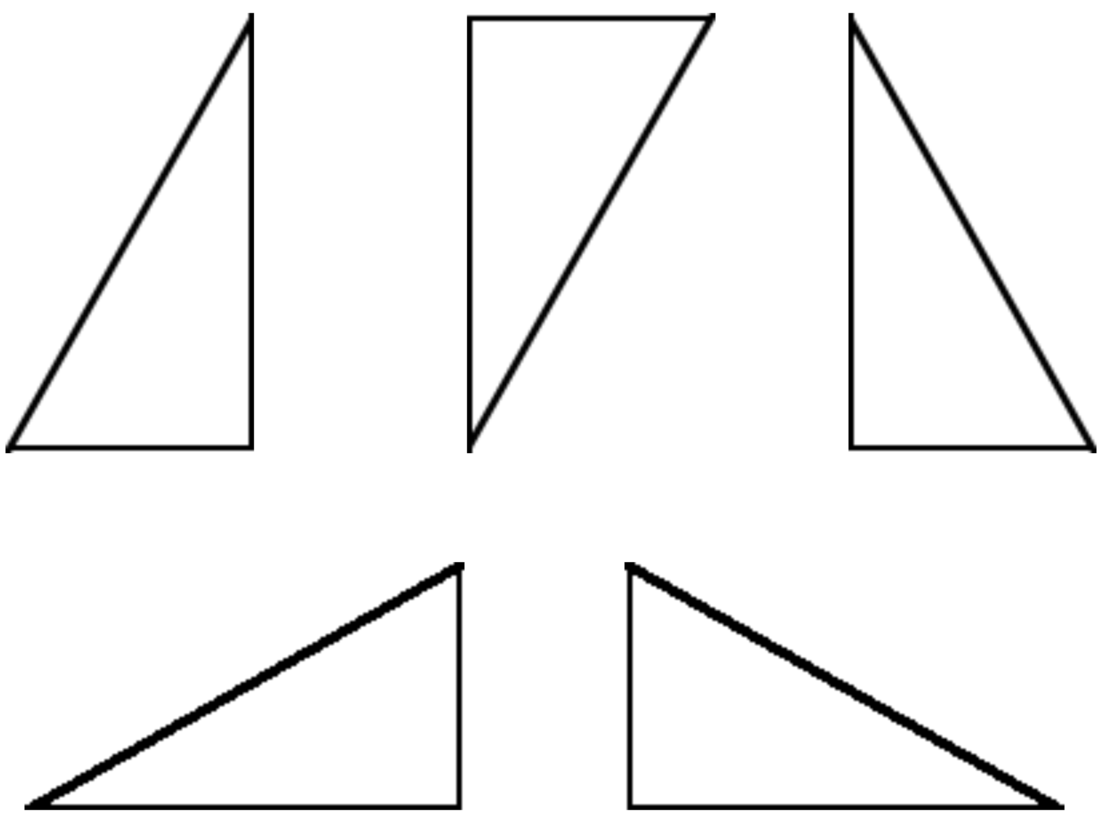
Sam Loyd's Juggler Puzzle

After juggling with the five right triangular pieces of cardboard to attract attention the clown proceeds to cut one of them into two pieces. He then lays the six pieces upon the top of the box and shows that they will fit together and form a perfect square.

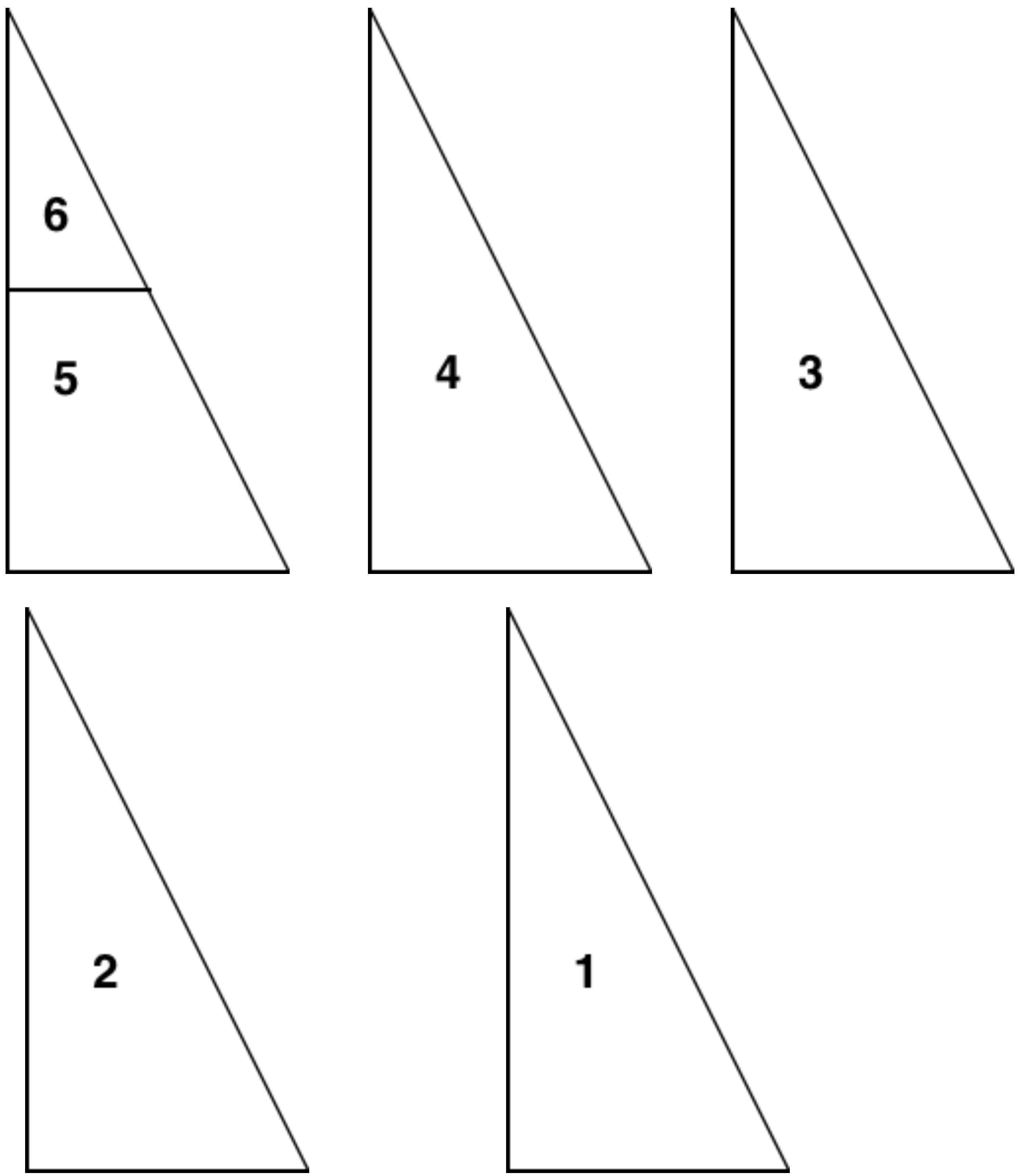
This puzzle first appeared in1914 in the Book <u>Sam Loyd's Cyclopedia of 5000 puzzles</u>. This collection has many of his most famous puzzles. A free downloadable version can be found at mma.org Mathematical Association of America

Samuel Loyd (1841 – 1911). Loyd is widely acknowledged as one of America's great puzzle-writers and popularizers.Martin Gardner called him "America's greatest puzzler". He published many versions of the geometrical vanish. One of Loyd's most notable puzzles was the "Trick Donkeys"

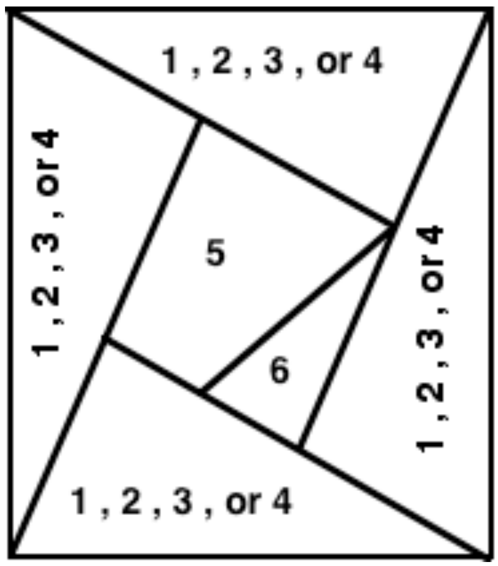


5 Right Triangles to a Square

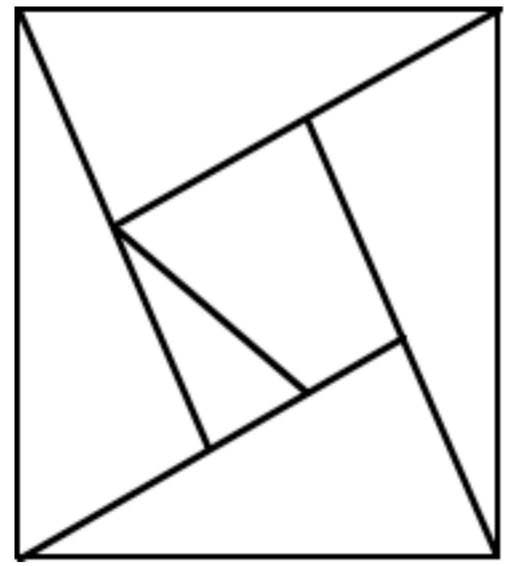
Cut out the 5 right triangles below. Cut the first right triangle into 2 parts along the line shown. Use these 2 parts and the 4 right triangles to form a square. Keep the numbered parts facing up.



Solution with the numbered parts facing up.



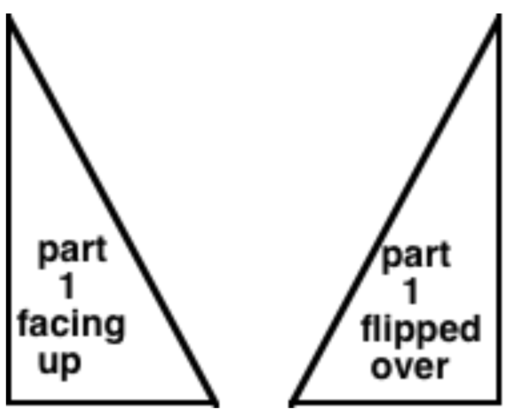
Solution with the numbered facing facing down.



Notice that the two solutions are not exactly the same. On solution is a reflection of the other.

Reflection Symmetry

Reflection Symmetry, sometimes called Mirror Symmetry, is easy to recognize, because one part is the reflection of the other part.   If you flip an object over the new shape is a reflection of the other one. It may seem like the two shapes would be the same but that may not be the true.



The two right triangles on the left are reflections of each other. Triangle 1 has not been rotated or slid around (translated) to get the second triangle. The first triangle hasbeen **REFLECTED (flipped over)** to get the second triangle. The two triangles may seen the same but there is an important difference. If one of the triangles fits into a position in a puzzle then the other one **CANNOT** take its place. One is “right orientated” and the other is “left orientated”.

Your own hands have reflection symmetry. Hold your your hands out in front of you and you will see that one is the reflection of the other. You cannot lay your right hand on top of your left hand and have them match. To have them match you need to flip one hand over.

Some shapes have reflection symmetry but both sides can be used to fill a position in the puzzle. Squares, rectangles, circles and equilateral triangle can be “flipped over and still work in a puzzle position. In most puzzles at least one part will not work this way so all the parts must face up. For this reason it is important to number the faces or use colored faces so the student is not confused.

If some parts face up and others face down an advanced student will see the need to reflect some of the parts to solve the puzzle. This is too frustrating for must students.

Teacher Notes:

I have provided a copy with the pieces numbered if you want to have your students use numbered parts. Some puzzles require that the “front of the pieces” all face up or the puzzle cannot be completed. In those cases you must have a way of keeping all the same sides facing up. Numbering is an easy way to do this. In this puzzle all the parts must face up or all the parts must face down. The puzzle can be solved either way but it cannot be solved with a mixture of fronts and backs. The concept of reflection is important to many geometric figures and you may want to discuss this as you work with any puzzle.

Class management:

The easiest way to introduce puzzles into your classroom is to use them as a “sponge” activity. Students can work on them when they have extra time. They may also work on them before or after class. To make this easy for you I suggest you have part of one bulletin board that is your “puzzle corner”. Students will know that 1 or 2 puzzles can be found on the bulletin board in that location.

Have the parts already cut out and placed in an envelope with the puzzle title on it. You may have 3 or 4 sets in the bag for most puzzles. Put the envelops and a set of answer recording sheets in a gallon size ziplock bag. Staple the bag (with the top open) to my puzzle board. Staple the puzzle tittle and instructions above the bag. When a student wants to work on the puzzle they read the instructions on the board. They take the puzzle and the recording sheet, solve the puzzle, record the solution and put the puzzle back in the bag. They give me the recording sheet. When you get time record the students name on a puzzle log that you post on the Puzzle Corner board. The log lets everyone see who has completed the puzzle. It acts as an incentive.

It sounds like a lot of work but it is not. Just take out a puzzle bag from your collection and staple the bag and the puzzle title on the board and announce there is a new challenge posted in the puzzle corner. After that all you need to do is record their name on the log. After a week, change out the puzzle for another one in my puzzle box. I would try to aim for using about 20 puzzles a semester.

The trick is to get a collection of puzzles that are at the correct level for your students and are easy to copy and cut out. Start with easy puzzles at the first of the school year and work towards harder ones as the year goes on. I prefer math related puzzles because they help support my curriculum but I also use others that have been proven to be popular. I hope my site supports your efforts.

5 Right Triangles to a Square

Instructions:

1. **Take the envelop and 1 recording sheet out of the bag.**
2. **Use the parts in the envelop to form a square.**
3. **Use the recording sheet to draw a picture of the solution.**
4. **Hand in the sheet to the teacher.**
5. **Put the 6 parts back in the envelop and put the envelop back in the bag.**

5 Right Triangles to a Square

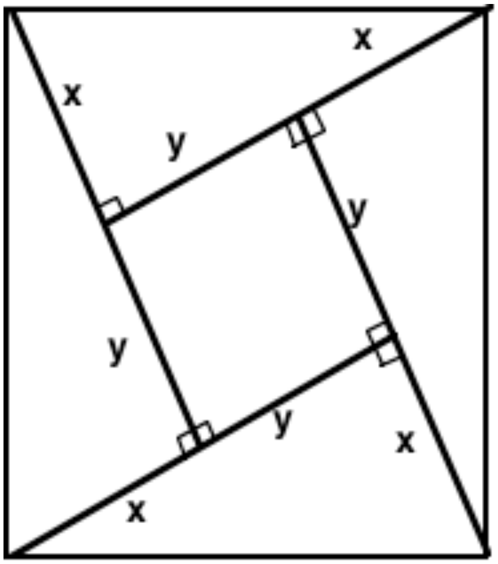
Recording Sheet:

Name ____________________

After you have formed a square with the 6 parts draw a picture of the solution in the space below. Be neat and clear. Put the 6 parts back in the envelop and put the envelop back in the bag so others can have a chance to solve the puzzle.

The use of algebraic and geometric techniques help to analyze the puzzle

When four right triangles are used to make a square they form square "hole" in the center.



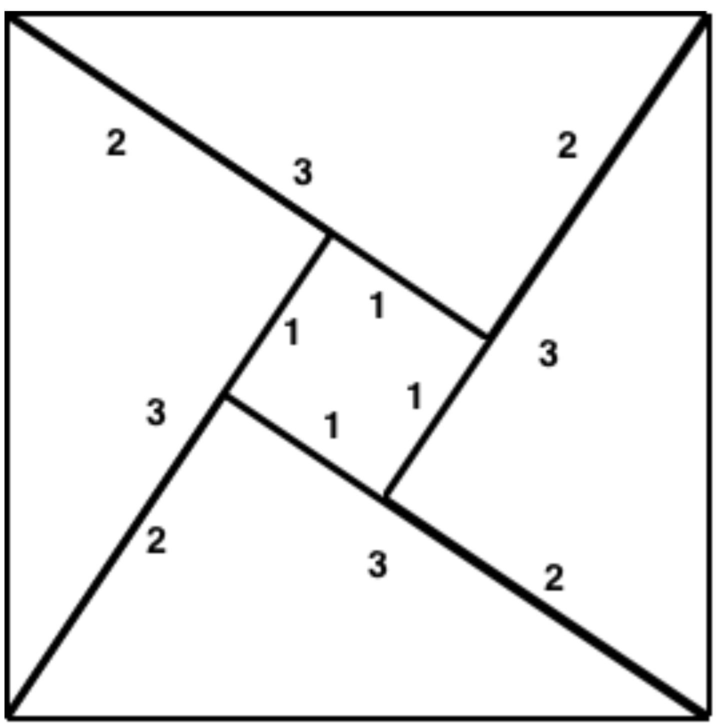
When 4 right triangles are used to form a square the "hole" in the middle must be a square.

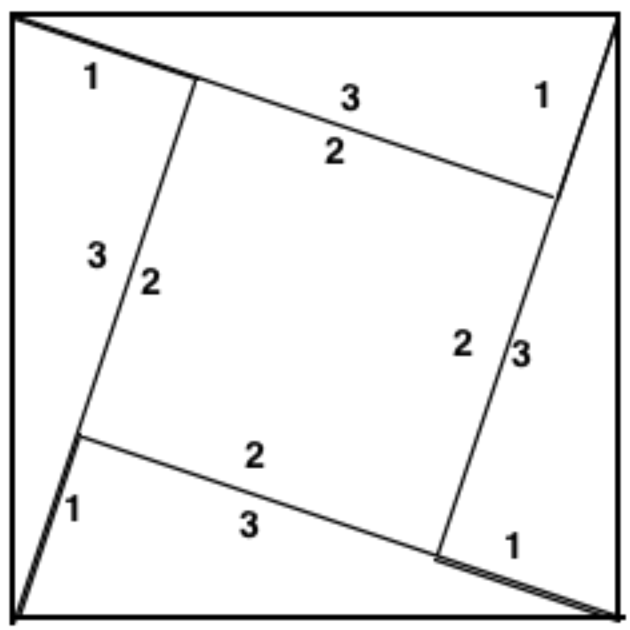
Call the base of each triangle x. The picture shows that the base of each triangle makes up part of the adjacent triangles height. It also shows that a right triangle is formed where they meet.

Call the remaining part of the triangles height y. Each triangle has a base of x and a height of x + y as shown.

The "hole in the middle always has 4 right angles and equal sides of length y so it is a square square.

The size of the square in the middle depends on the length of the base and height of the triangle. Two examples are shown below. The triangle on the left have a base of 2 and a height of 3. The triangles on the right have a base of 1 and a height of 3.



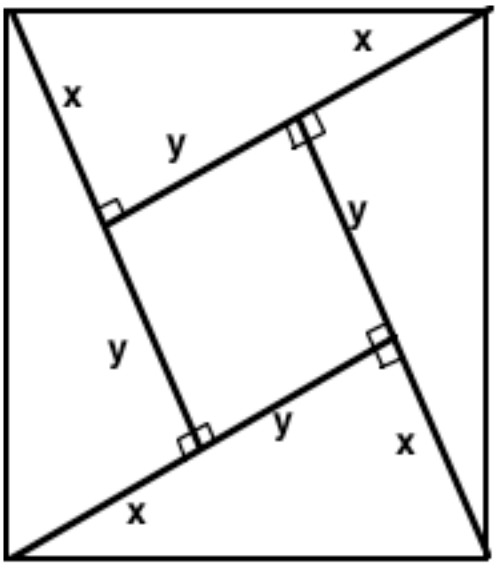


The solution requires that 4 of the right rectangles form a square. We have proven that this will always happen if you place the triangles as shown in the picture.

The solution also requires that the hole be filled by the 5th right triangle given in the puzzle.

This will NOT happen with all right triangles.
The ratio of the base to height for the right triangles must be 1 to 2

The solution requires that the square in the center be filed by the 5th right triangle given in the puzzle. This mean that the square and the triangle must have the same area,
The area of the square is y^2 and the area of the right triangle is $\frac{1}{2}(x)(x+y)$. These areas must be equal. Setting the area equal to each other lets us find the relationship between x and y.

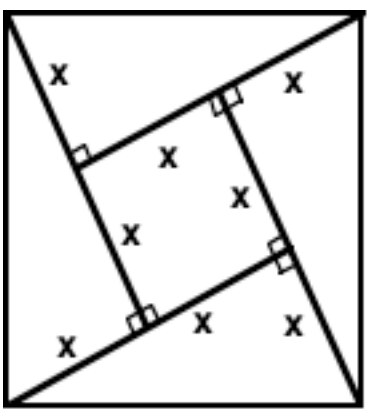


$$y^2 = \frac{1}{2} \bullet x \bullet (x+y)$$
$$2y^2 = x^2 + xy$$
$$2y^2 - xy - x^2 = 0$$
$$(2y+x)(y-x) = 0$$
$$2y + x = 0 \quad \text{or} \quad y - x = 0$$

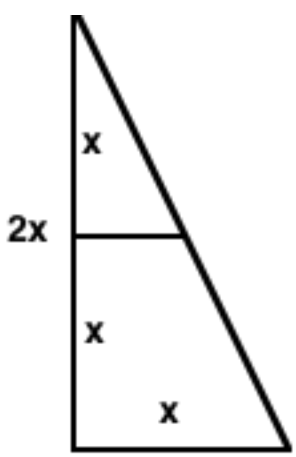
$$x = \frac{-1}{2y} \quad \text{or} \quad y = x$$

x and y must bepositive so $x \neq \frac{-1}{2y}$

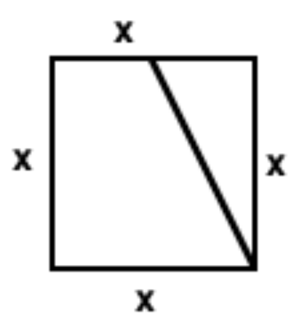
so $y = x$



If x = y then the right triange has a base of x and a height of 2x .



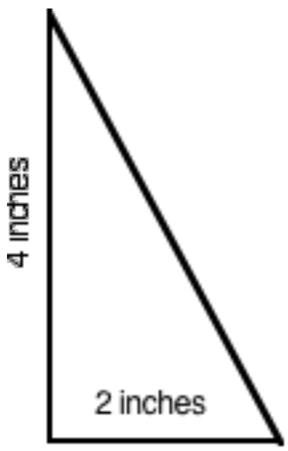
The top part of the triangle is then rotated to match the bottom part and make a small square with an area of x^2



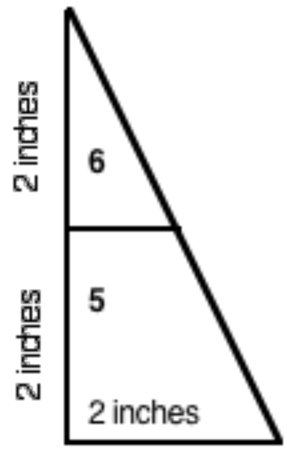
This is the square we need to fill the hole in the middle

The only right triangles that will work for this puzzle
are right triangles with a ratio of the base to height of 1 to 2

The triangle provided for the puzzle are right triangles with a base of 2 inches and a height of 4 inches. **The ratio of the base to height for the right triangles is 1 to 2.**

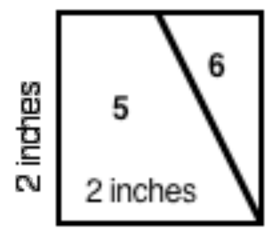


The triangles used in this puzzle have a base and height that are in a ratio of 1 to 2

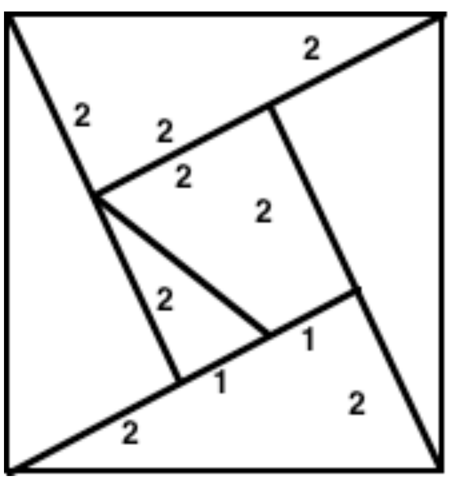


The triangle is cut with a line that is parallel to the base and 1/ 2 from the base to the top.

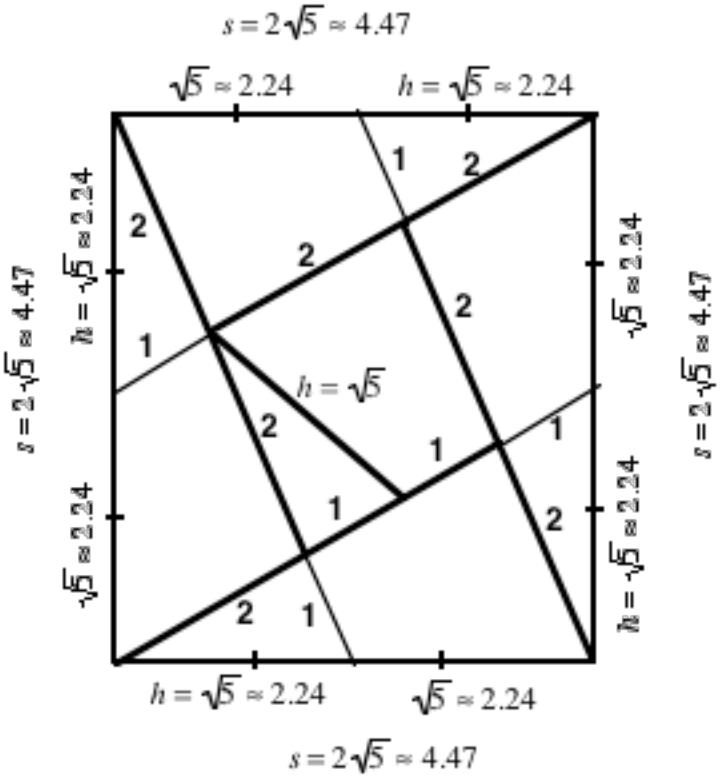
The top part of the triangle is then rotated to match the bottom part and make a small square



The small square fits in the middle of the 4 large triangles.



Geometric and Trigonometric Relations:



$h^2 = 1^2 + 2^2$

$h^2 = 1 + 4$

$h^2 = 5$

$h = \sqrt{5}$

$h \approx 2.24$

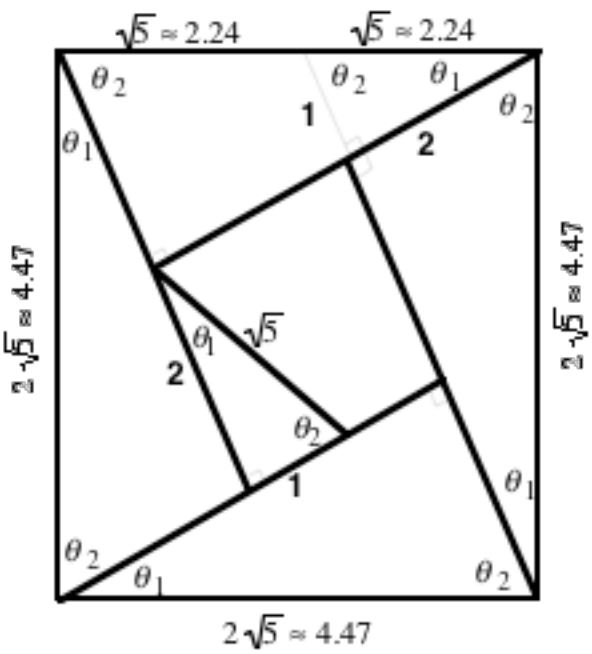
$s^2 = 2^2 + 4^2$

$s^2 = 4 + 16$

$s^2 = 20$

$s = \sqrt{20} = 2\sqrt{5}$

$s \approx 4.47$



$tan(\theta_1) = \frac{1}{2}$

$\theta_1 = \arctan\left(\frac{1}{2}\right)$

$\theta_1 \approx 26.6°$

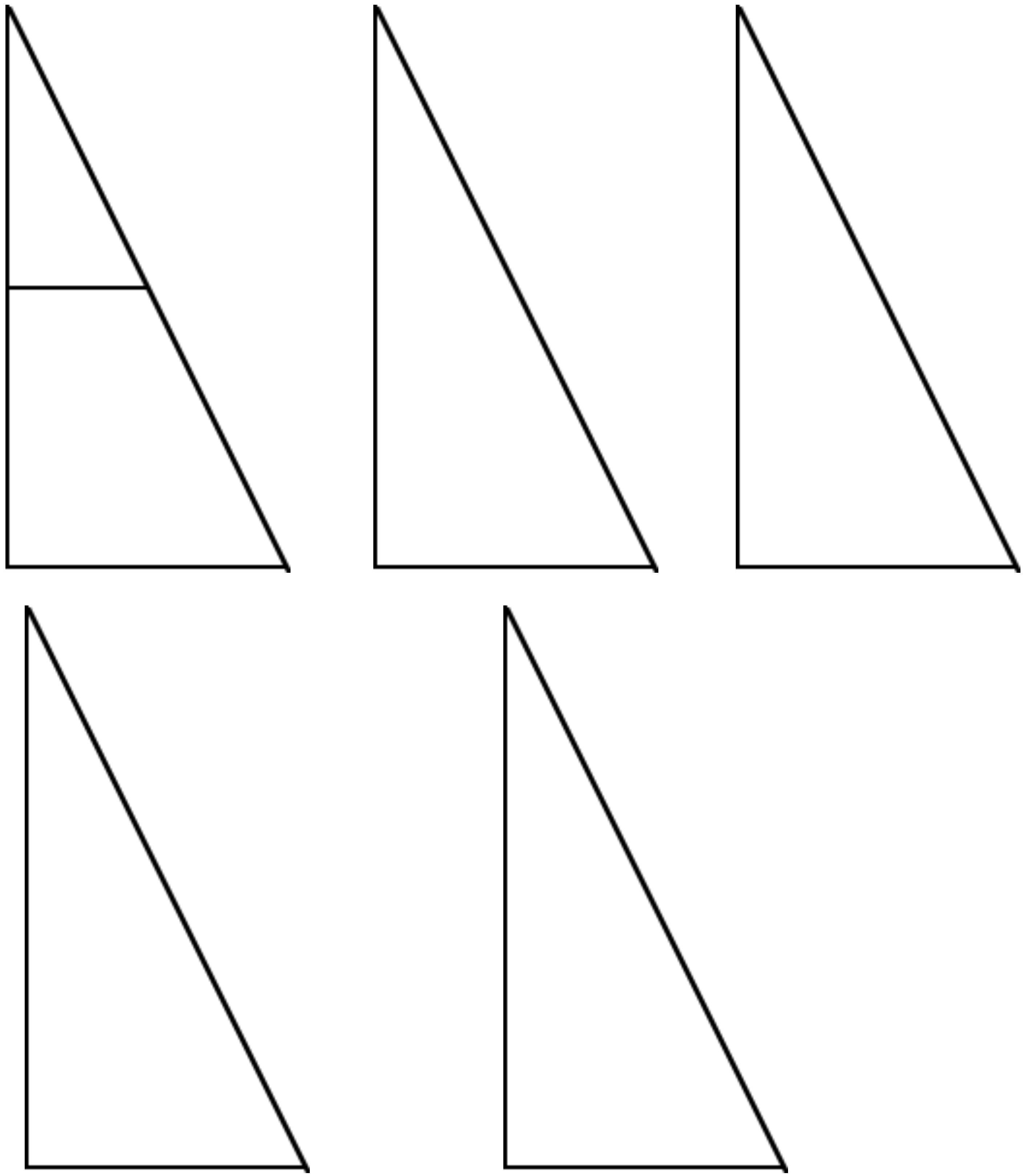
$tan(\theta_2) = \frac{2}{1}$

$\theta_2 = \arctan\left(\frac{2}{1}\right)$

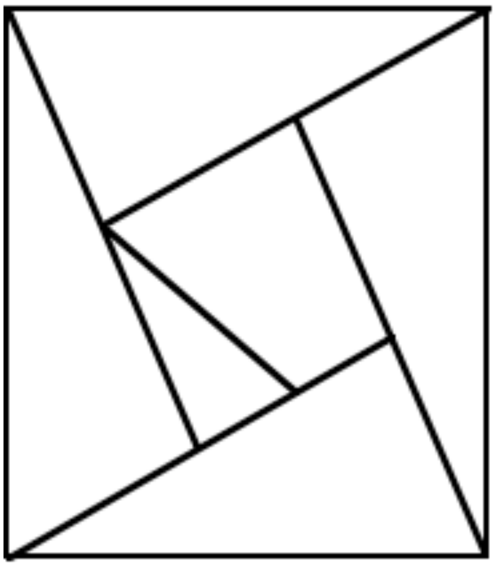
$\theta_1 \approx 63.4°$

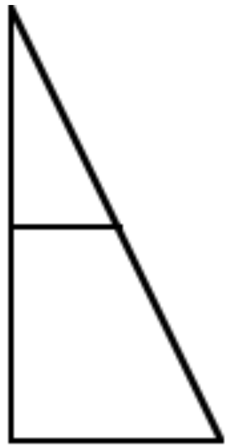
5 Right Triangles to a Square

Cut out the 5 right triangles below. Cut the first right triangle into 2 parts along the line shown. Use these 2 parts and the 4 right triangles to form a square.



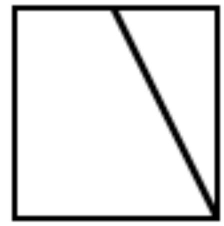
Solution with the numbered parts facing up.





The triangle is cut with a line that is parallel to the base and 1/ 2 from the base to the top.

The top part of the triangle is then rotated to match the bottom part and make a small square



The small square fits in the middle of the 4 large triangles.