The 15 Puzzle

As pictured below the 15 Puzzle has 15 numbered wooden tiles in a 4x4 box, with one empty space in the bottom right corner. The object was to rearrange the 15 pieces in any order and then use the empty space to slide the tiles back into the original numerical order. When the puzzle appeared around the year 1870 it created as big a craze as the Rubik's Cube did a hundred years later.

In later versions it was called the GEM puzzle

The FAD of the 1870's

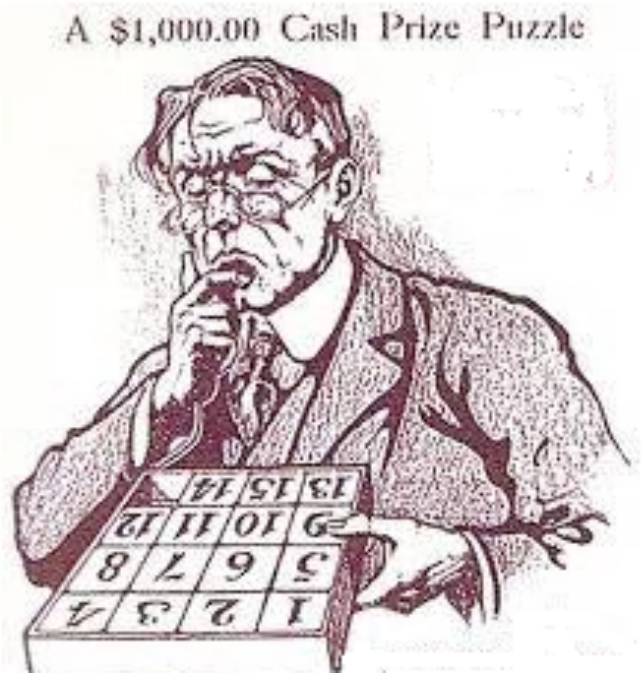
Sliding puzzles started with a bang in the late1800's. The Fifteen Puzzle started in the United States around 1870 and it spread quickly. Owing to the uncountable number of devoted players it had conquered, it became a plague. In a matter of months after its introduction, people all over the world were engrossed in trying to solve what came to be known as the 15 Puzzle. It can be argued that the 15 Puzzle had the greatest impact on American and European society in 1880 of any mechanical puzzle the world has ever known. Factories could not keep up with the demand for the 15 Puzzle. In offices an shops bosses were horrified by their employees being completely absorbed by the game during office hours. Owners of entertainment establishments were quick to latch onto the rage and organized large contests. The game had even made its way into solemn halls of the German Reichstag. "I can still visualize quite clearly the grey haired people in the Reichstag intent on a square small box in their hands," recalls the mathematician Sigmund Gunter who was a deputy during puzzle epidemic. In Paris the puzzle flourished in the open air, in the boulevards, and proliferated speedily from the capital to all the provinces. A French author of the day wrote, "There was hardly one country cottage where this spider hadn't made its nest lying in wait for a victim to flounder in its web." In 1880 the puzzle fever seems to have reached its climax.

The start of the craze

This first version of the 15 puzzle was invented by an upstate New York postmaster named Noyes Chapman. He is said to have shown friends, as early as 1874, a precursor puzzle consisting of 16 numbered blocks that were to be put together in rows of four, each summing to 34. Copies of the improved Fifteen Puzzle made their way to New York by way of Noyes' son Frank and from there to Hartford Connecticut, where students in the American School for the Deaf started manufacturing the puzzle and selling them in Boston Massachusetts. Shown one of these, Matthias Rice, who ran a fancy woodworking business in Boston, started manufacturing the puzzle in December 1879 and convinced a "Yankee Notions" fancy goods dealer to sell them under the name of "Gem Puzzle".

The "Fifteen" Puzzle consists of the same blocks or figures, with the figure 16 taken out as on the plan; leaving one blank space so as to give room to move the Blocks. After well mixing the figures place them in the box, and move them without taking them out until they are numerically in order.

Sam Loyd and Cash helped cause the chaos



The most popular version was due to the famous puzzler Sam Loyd, who presented all the tiles in order, except with the 14 and the 15 swapped. This became known as the 14-15 puzzle. Sam Loyd asked the editor of a New York newspaper to publish the puzzle in the Sunday edition. with a reward of $1,000 for its solution. Loyd offered a prize of AUS $1000, worth about $20,000 today, to the first person to solve his puzzle. People all over the country tried to win this grand sum, but none succeeded. To win the prize you had to start start with the 1 to 13 squares in order and the 14 and 15 squares out of order as shown above. What people didn't know was that only half of all possible starting arrangements of pieces can be solved. This starting arrangements was one of the starting positions that cannot be solved. That made it safe to advertise the $1000 prize.

Sam Loyd claimed from 1891 until his death that he invented the puzzle, writing in the Cyclopedia of Puzzles "The inhabitants of Puzzle land will remember how in the early seventies I drove the entire world crazy over a little box of movable pieces which became known as the '14-15 Puzzle'. However, Loyd had nothing to do with the invention or initial popularity of the puzzle,

Sam Loyd came to be widely known as an author of amusing problems and a multitude of puzzles. Curiously enough, he failed to patent his Fifteen Puzzle in the USA. According to the regulations, he had to submit a '"working model" so that a prototype batch could be manufactured from it. He posed the problem to a Patent Office official, but when the latter inquired if it were solvable, the answer was 'No, it's mathematically impossible'. The official therefore reasoned: "n which case there can't be a working model and without a working model there can be no patent." Loyd was satisfied with the decision.

Puzzle poems are very rare. But there are many poems about the 15 puzzle.

Here are 3 examples

The Gem Puzzle.

Full many a gem-the puzzle you have seen-
The dark, unfathomed caves of lying bare;
Full many a sorely puzzled wight, I ween,
Will lose his patience and will loudly swear.

But who, unto dumb foolishness a prey,
This pleasing, anxious puzzle e'er resigned
When he would lie, "I did it the other day,
But how, I cannot now quite bring to mind?"

Nor you, ye proud, impute to these the fault
If never any one believe their tales:
You know full well they'd scarcely earn their salt
By puzzling o'er it till their reason fails.

No further seek their lying to upbraid.
Or ask them how they did the blasted thing;
The puzzle 't is that liars them has made,
The wicked swear-words to their lips does bring.

Boston Transcript, February 12, 1880.

One More Unfortunate

She couldn't get the puzzle solved,
For lack of time-a minute,
And if you would avoid her fate,
Why, never you begin it.

Evening Gazette; Jervis Post, NY, March 11, 1880

Push, Brothers, Push

Push, brothers, push with care,
Push the 14 to the 15 square;
The 6 to the 7, and you've got it there;
The 10 to the 9, or you don't care where,
But the 15 and the 14 they will stick there.
Push, brothers, push with care
Till your minds are all a jumble and you tear your hair.

World; Utica Morning Herald, March 2, 1880.

Squares to Rectangles

The next development in sliding puzzles came about in 1909 when Lewis W. Hardy invented the first sliding puzzle using pieces that were rectangular rather than square. He called it the Pennant Puzzle, giving it a baseball theme. It was also known as Dad's Puzzle. Ma's Puzzle, which followed in 1927, introduced two L-shaped pieces, making it even more difficult to solve.

Sliding Puzzles

The original puzzles had 15 wooden pieces that you could pick up and move to any location. You slid the pieces after the original setup. Half of the layouts created will not have a solution. A more modern version put slots and groves in the sides of the movable pieces so that you not could pick up a piece and move it to any location. It made the puzzle portable and easy to put in your pocket and play when you found time. It also locked in the layout. This had the advantage of putting the pieces in an order so that the puzzle could always be solved. It ended the $ 1000 prize contests but it became a large hit anyway. You slid the squares in the frame. The arraignment you found the puzzle in was where you started. From that arraignment you slid the numbers until the correct 1 to 15 order was achieved.

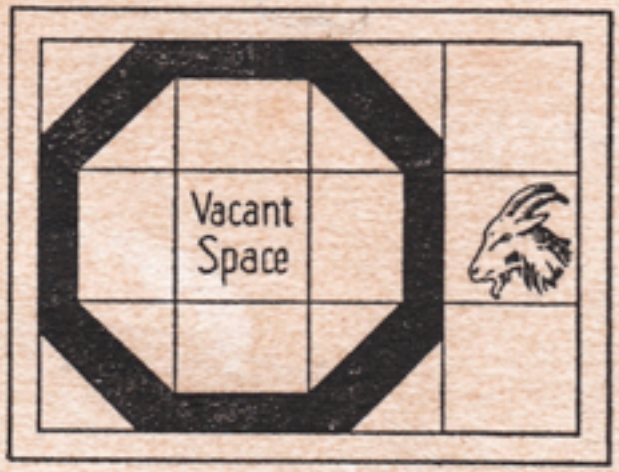
There are hundreds of versions on the market today. Many use pictures instead of numbers.

Some versions had you rearrange letter to spell out a sentence.

Over the years, there were many novelty variations on the sliding puzzle. "Capture the Kaiser," featuring pictures of the German ruler, came out during World War I. Get My Goat was a copy.

SOLUTION OF

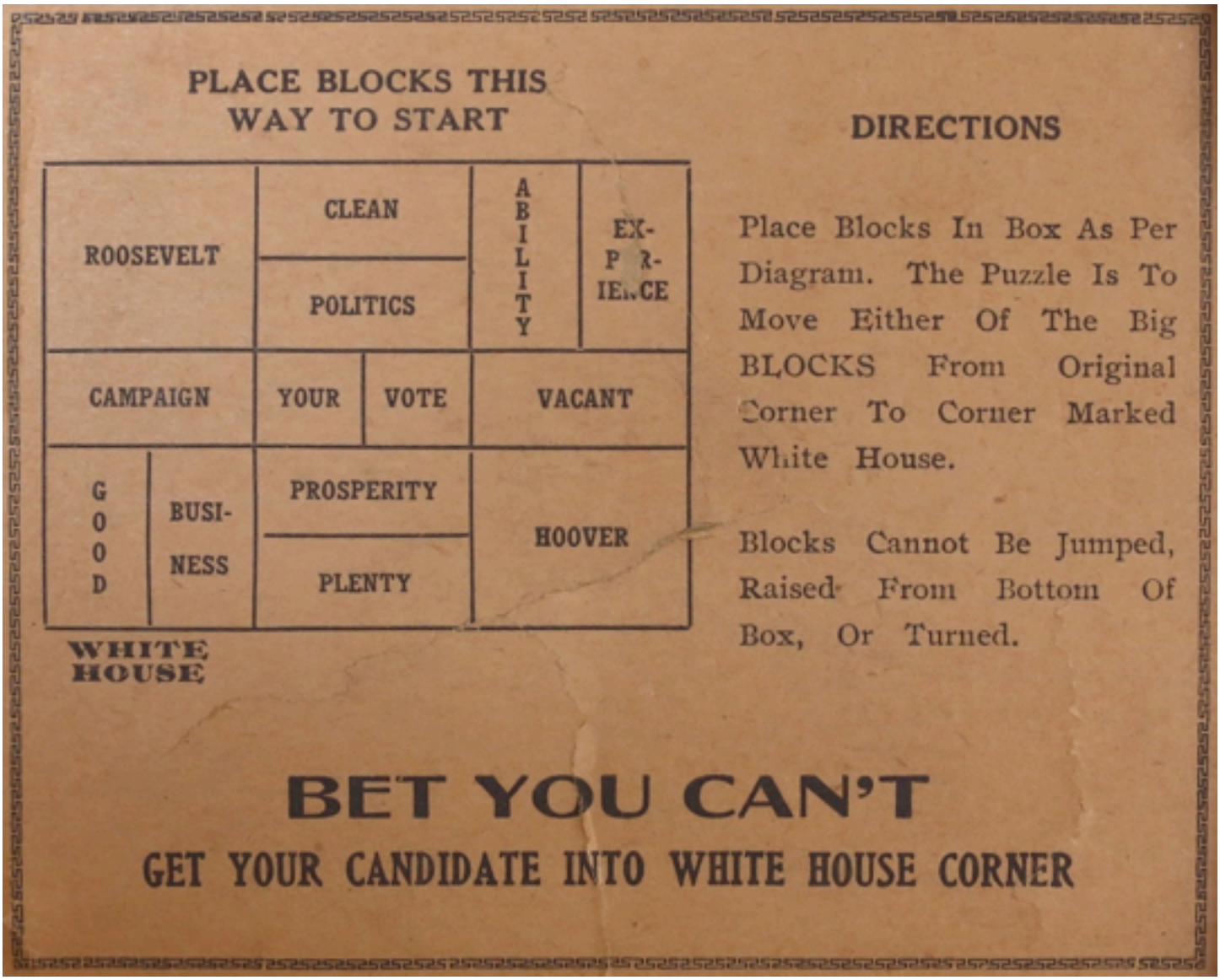
Get my Goat Puzzle



START

Move block in front of goat to left: 1 up, 2 to right, 2 down, 1 to left, 2 up, 1 to right, 2 down, 1 to left, 2 up, 1 to right, 2 down, 1 to left, 1 up, 1 to left, 1 up, 2 to right, 2 down, 1 to left, 2 up, 1 to right, 2 down, 1 to left, 1 up, 2 to left, (Goat) 1 up (Blank one,) 2 to right, 1 down, 1 to left (Goat,) 1 up, 1 to left.

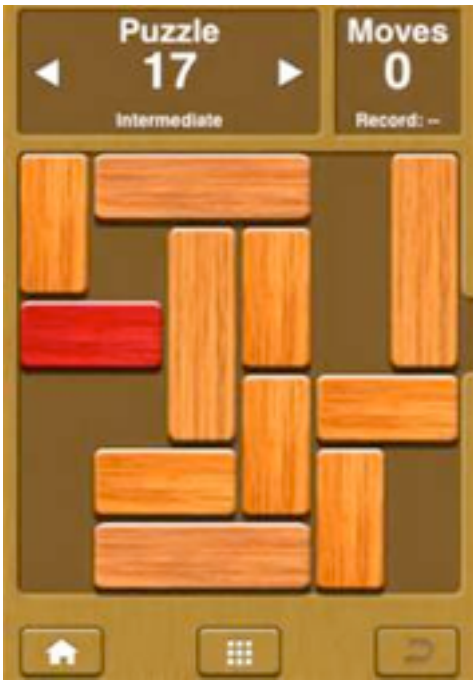
The 1932 presidential election campaign between Herbert Hoover and Franklin Roosevelt was commemorated in a slider.

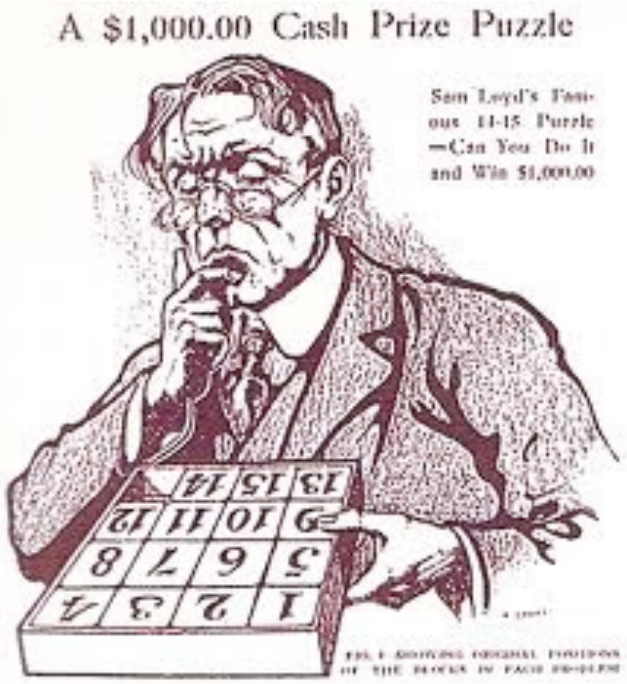


A U.S. Political cartoon about finding a Republican presidential candidate in 1880

A puzzle from 1934 called "Line up the Quinties" had players sliding squares with pictures of the famous Dionne quintuplets.

Since the computer began to play a role in designing and solving puzzles, the variety of puzzles has increased. Hundreds of pictures have been transformed into sliders, and increasingly more difficult arrangements of geometric shapes have challenged solvers. A web search of sliding puzzles will produce many web sites where you can play the puzzle online.





The picture to accompany the 14-15 puzzle in Sam Loyd's Puzzle book


LITTLE
BUTTERCUP
PUZZLE.
DIRECTIONS.
IT LOOKS EASY—TRY IT.
L I T T
L E F U
Z Z E
P

"The Game of Fifteen Gem Puzzle", manufactured by Alan L. Lovejoy, Boston, 1880.

"The Boston Puzzle", circa 1880's.

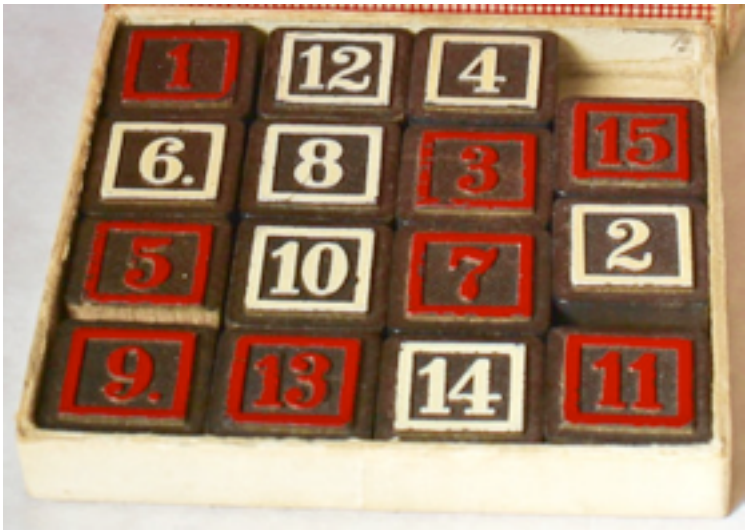

1 2 3 4
5 6 7 8
9 10 11 12
13 14 15
THE POPULAR
15
PUZZLE
BRITISH MANUFACTURE

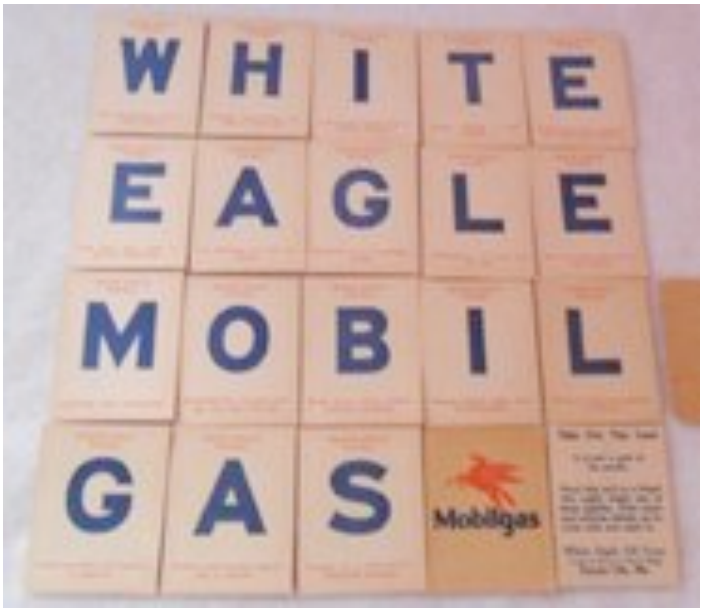

1 2 3
4 5 6 7
8 9 10 11
12 13 14 15
THE CROSS NUMBER
"15"
PUZZLE
DIRECTIONS
START
FINISH
Price - 25 cents
The Cross Number 15 Puzzle
Room 915 470 Main St.
FITCHBURG MASSACHUSETTS
PAT. APPLIED FOR

You No Can Do !


1 2 3 4
5 6 7 8
9 10 11 12
13 14 15
"UNOCANDO"
FOR CHILDREN AND GROWNUPS
HA-HA
HA-HA
COPYRIGHT 1946
GOETZE GAMES COMPANY


15 • PUZZLE


1 12 4
6. 8 3 15
5 10 7 2
9. 13 14 11


WHITE
EAGLE
MOBIL
GAS
Mobilgas


The 15 PUZZLE
MOVE
THE NUMBERS
(WITHOUT LIFTING)
UNTIL ARRANGED IN THE ORDER 1-15
BRITISH MADE


PROSPERITY
OUR
HOME
LAND
PUZZLE
25¢
USA
OUR
HOME
LAND


SLIDING
BLOCK
PUZZLE
1
2
3


SLIDE-A-PUZZLE
P I G
C O W
C A T
D O G


Fifteen Puzzle
Known the
Fifteen Puzzle
world over!


Thinkfun
Everybody Plays
Fifteen Puzzle
Stainless Steel Construction
The Fifteen Puzzle has resurfaced many times over the years, often with some small twist or claim of originality.
Ours is a faithful reproduction of one originally called "The IMP," which was introduced in 1933.
Its smoothly sliding stainless steel tiles with decorative enamel facings represent the purest, highest quality rendering of this classic that we've ever seen.
Handy Travel Case
Instruction Book With Over 30 Challenges
Fifteen Puzzle


PUZZLE map of U.S.A.
A GAME OF KNOWLEDGE AND SKILL FOR ALL AGES
29
NO. 1056A
MADE IN HONG KONG


SLIDE-A-PUZZLE
P I G
C O W
C A T
D O G

One unusual variant was Genius by Milton Bradley. This is an eight tile 3×3 puzzle. The twist is that the tiles are identical, and that there is an LED light under each. The aim is to slide the tiles to make all their LEDs light up. Another amusing variation is called Virus, and it has a big spider-like bug with each of its six legs stuck to a tile of a 4×4 grid. The legs restrict some of the movement of the tiles.

The Metropolitan Museum of Art's Color Magic Puzzle is a beautiful version. It is large and heavy, with felt underneath, intended as a display piece on a table. The tiles are colored transparent plastic, and the base behind it has colored squares. The squares in the base come in three colors, red, blue, and yellow, alternating in that order all the way through, except for the sixteenth square which is grey. The tiles are also red, blue, and yellow, five of each. At the start, every tile is on a square of the same color, so that the puzzle shows diagonal lines of the primary colors red, blue, and yellow. The aim is to mix the tiles so that the diagonals now show the secondary colors orange, green and purple. This can be done in two ways (orange can be a yellow tile on a red square or a red tile on a yellow square, etc).

Notes to the Teacher

This is an historical puzzle. In the 1870's many people would have been given this puzzle as part of an advertising campaign. It was common to use puzzles for this purpose. The great puzzle designer Sam Loyd got involved with the puzzle by devising an advertising campaign based on a $ 1000 prize. Of course his version could not be solved so he took no risk.

The modern version is a simple plastic tray with the numbers locked in the tray with slots and groves in the sides of the pieces. The intent was to have the person try to solve the puzzle without help. After they became frustrated the person could then look at the instructions. They would then use then use the solution provided and try to follow the numerous steps required to solve the puzzle.

THE INTENT OF THE PUZZLE IS NOT TO HAVE A STUDENT SOLVE IT. THE INTENT IS TO HAVE THE STUDENT EXPERIENCE FOLLOWING LONGER TECHNICAL INSTRUCTIONS STEP BY STEP.

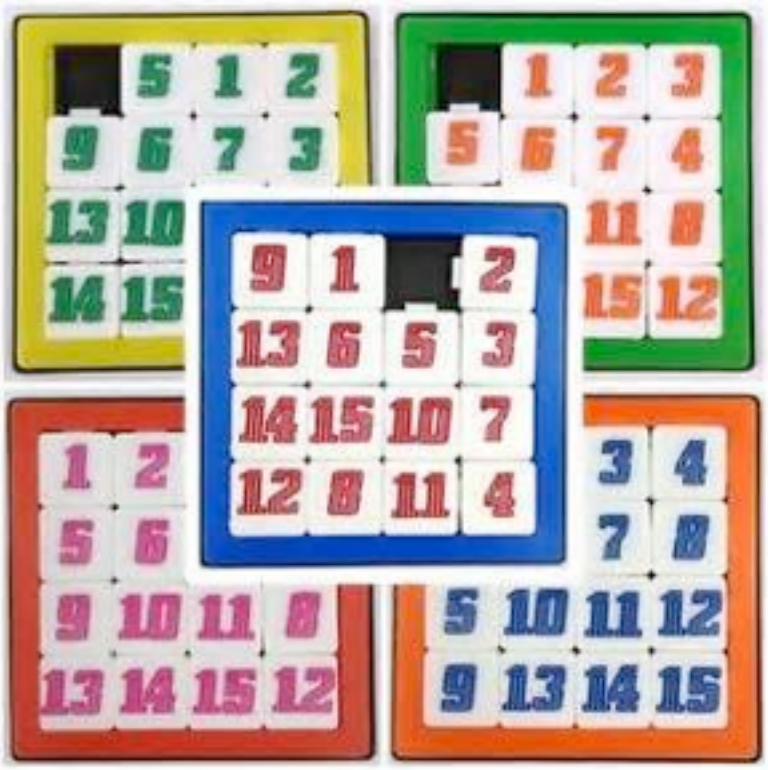
The skill of being able to follow a complex solution to complete a task or project is a skill that is very much in demand in by employers in almost every area of today's work place. The jobs that pay well often require the person to follow more complex steps to complete a task then in the past. This puzzle from the past is a great starting point to help students experience following complex instructions.

Many student want to be given easy problems that can be completed in one or two step steps. The mathematics or science they experience in the basic mathematics and science classes often do not require a long process. In the more advanced math and computer courses they experience later do require many steps and many students balk at the amount of work required or ask for a tool to do the work for them. Thats is understandable but they need to be pushed beyond that skill level.

When I use this puzzle with students I give them the puzzle and we talk about how the pieces slide. I show them the basic moves and I give them hints. I then let them try to get the number 1 to 8 in place. That is the first part of the solution. If they get that on their own I let that group try to get the next 2 rows in the correct position while I work with the students who need help with part one, I may use students who really understand the moves to solve part 1 to work 1 on 1 with students who need help.
When we have most of the students ready to solve the last 2 rows I repeat the process. I show them the basic moves and I give them hints. I then let them try to get the number 9 to 15 in place.

I then have them watch a u-tube video of the solution and try to copy the moves they see there and see if they can get the numbers in the final locations It may take a bit of help to gave them follow the instructions but with help they can do it, It is quite a feat for them to succeed. I want to remind you that almost every student can complete the if they focus and persist. These are the real skills that they are learning and the real value in problem solving in general.

These puzzles are cheap. It does not cost a lot to get a puzzles for every 2 students



Solve Row 1

You can always get the numbers 1 to 4 in the upper row. Do that first

A number of movements can always bring the 1 to the place occupied by it originally. In exactly the same way we can, without touching the counter 1 move the counter 2 to the adjacent place on the right. Next, without touching either the 1 or 2, we can move the 3 and 4 to their normal places. If these occasionally are not in the two last columns, we can bring them there and through a number of movements achieve the arrangement sought.

Solve Row 2

You can always get the numbers 5 to 8 in the second row. Do that next

Bring the second line in the normal order. Further, within the space of the two last lines we'll need to arrange the counters 9 and 13, which is always possible, too.

Solve Row 3 and 4 together

You can always get the numbers 10 , 11 and 12 in order in the third row. Do that next

Bring 10, 11, and 12 into the normal arrangement. You may need to switch some pieces between 3 and 4 so then when row 3 is solved row 4 is also solved.

The Fifteen Puzzle and the 34 Magic Square Puzzle

Old versions of the fifteen puzzle typically had pieces that could be removed, and sometimes a piece 16 was included that was not used to play the normal fifteen puzzle. The 16 could be left in to create another puzzle. It was not a sliding puzzle as there was no open space. This puzzle asked that you arraigned the 16 numbers into 4 rows of 4 numbers so the each row, each column and both diagonals add up to 34. This is the basic form of a Magic Square.

One example is the Boss puzzle or the "Thirty Four" problem.

This puzzle was titled as the 15 and 34 puzzles *De La Rue & Co., London, circa 1880*.

This puzzle was a 15 and 34 puzzle

THE GEM PUZZLES,

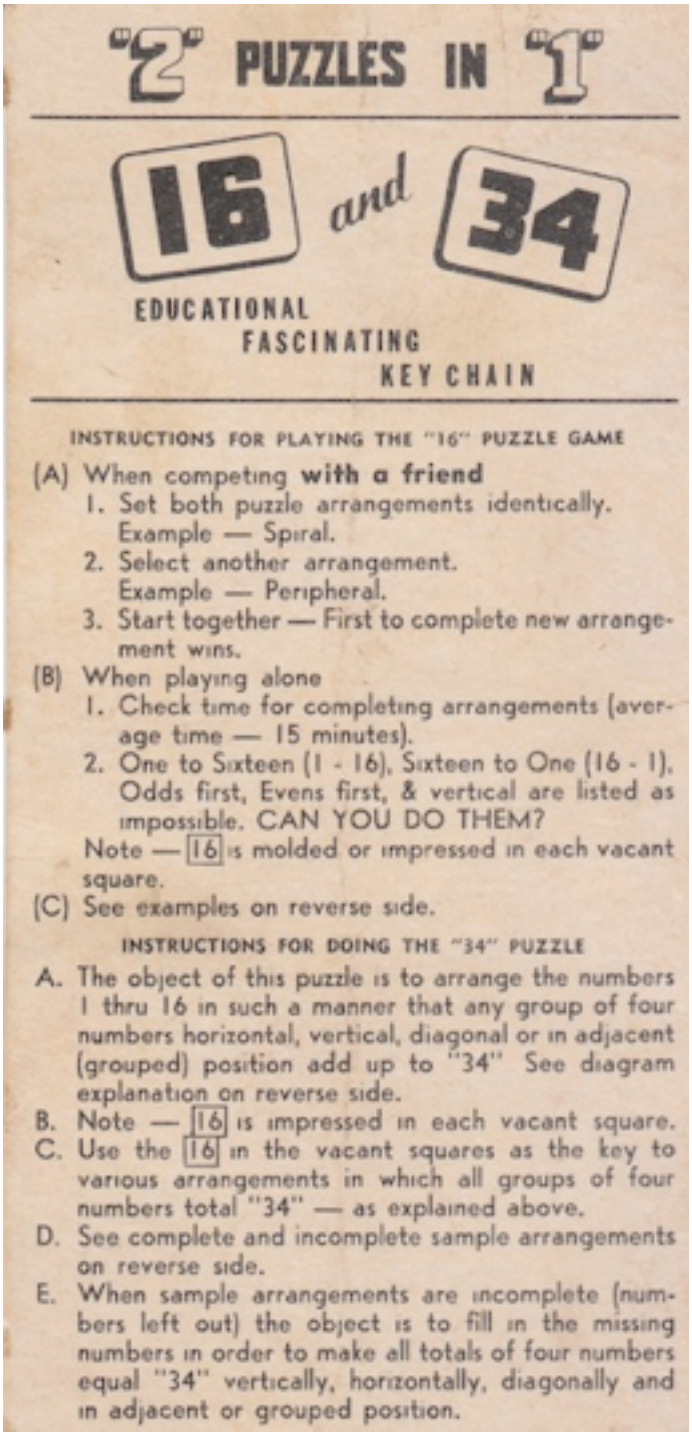
15 & 34, a most interesting combination of figures, both in one box.

The "Thirty Four" Puzzle consists of 16 blocks, numbered from 1 to 16, these are in a small square box, and the Puzzle is to so place the figures as to add up in 16 different ways, each to make Thirty Four, the addition must be the same counting from corner to corner, horizontally, perpendicularly, and the four blocks in each corner, also the four outside numbers of the two middle lines.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

The "Fifteen" Puzzle consists of the same blocks or figures, with the figure 16 taken out as on the plan; leaving one blank space so as to give room to move the Blocks. After well mixing the figures place them in the box, and move them without taking them out until they are numerically in order.

Many puzzles sold combined problems for fifteen and sixteen pieces; here are the front and back of directions that came with a "2 puzzles in 1" puzzle:



"2" PUZZLES IN "1"

16 and 34

EDUCATIONAL
FASCINATING
KEY CHAIN

INSTRUCTIONS FOR PLAYING THE "16" PUZZLE GAME

(A) When competing **with a friend**
1. Set both puzzle arrangements identically. Example — Spiral.
2. Select another arrangement. Example — Peripheral.
3. Start together — First to complete new arrangement wins.

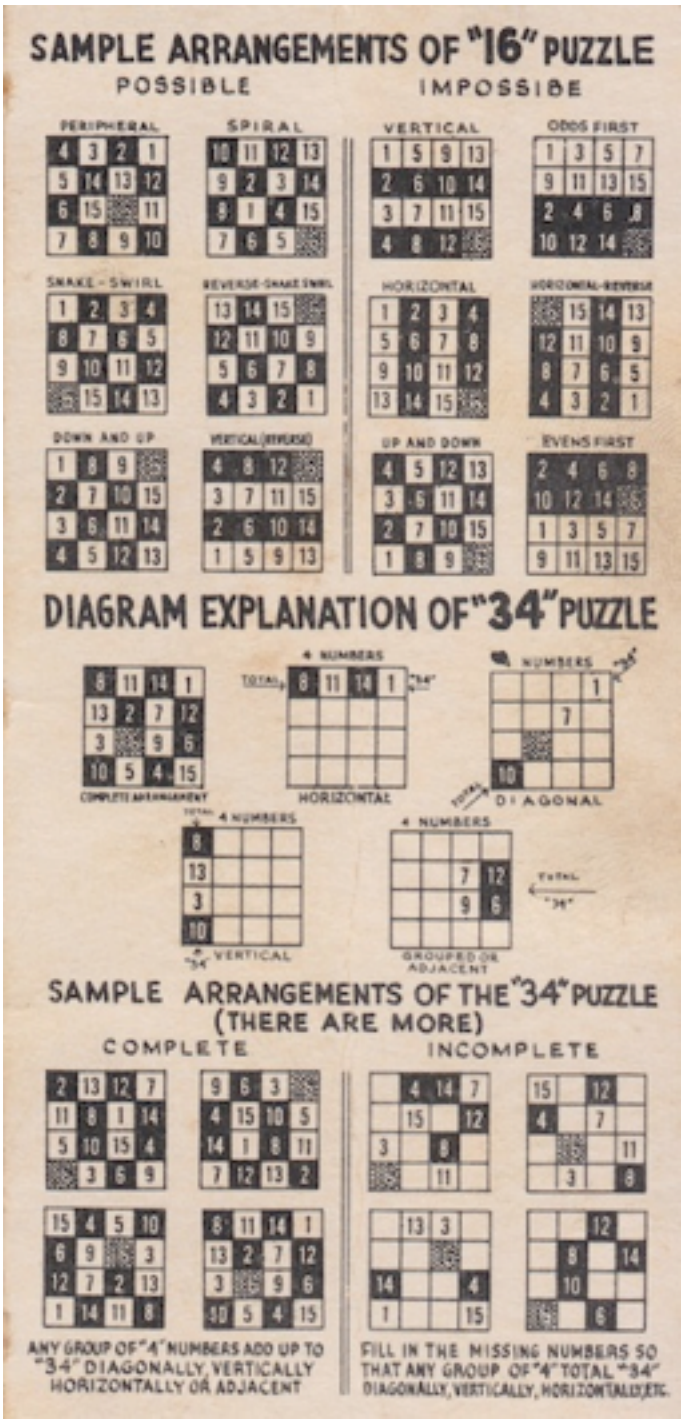
(B) When playing alone
1. Check time for completing arrangements (average time — 15 minutes).
2. One to Sixteen (1 - 16), Sixteen to One (16 - 1). Odds first, Evens first, & vertical are listed as impossible. CAN YOU DO THEM?

Note — 16 is molded or impressed in each vacant square.

(C) See examples on reverse side.

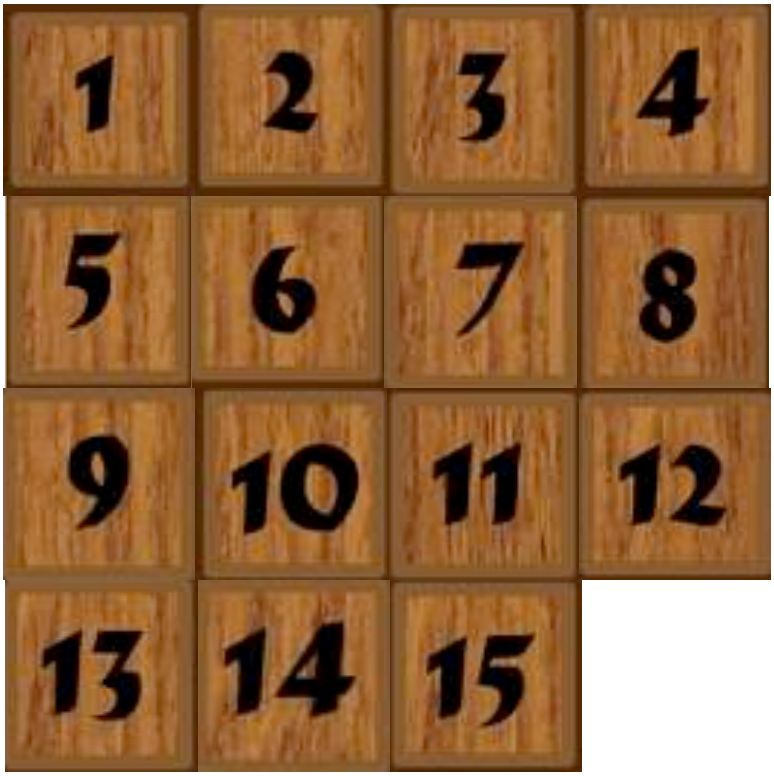
INSTRUCTIONS FOR DOING THE "34" PUZZLE

A. The object of this puzzle is to arrange the numbers 1 thru 16 in such a manner that any group of four numbers horizontal, vertical, diagonal or in adjacent (grouped) position add up to "34" See diagram explanation on reverse side.
B. Note — 16 is impressed in each vacant square.
C. Use the 16 in the vacant squares as the key to various arrangements in which all groups of four numbers total "34" — as explained above.
D. See complete and incomplete sample arrangements on reverse side.
E. When sample arrangements are incomplete (numbers left out) the object is to fill in the missing numbers in order to make all totals of four numbers equal "34" vertically, horizontally, diagonally and in adjacent or grouped position.



Many puzzles sold combined problems for fifteen and sixteen pieces

The Math Behind the 15 Puzzle



The numbers start in the box in this position

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

The numbers are rearranged in the box

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

The numbers end in the box in this position

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

"The challenge of the puzzle is as follows: using successive movements made possible by the presence of the blank space the **arbitrarily arranged squares** should be brought to the normal arrangement, i.e. the counters are in numerical order with the 1 in the upper left corner, followed by the 2 on the right, then the 3 and the 4 in the upper right corner; in the next row there should be from left to right the 5, 6, 7, 8, , the third row should be 9, 10 , 11, 12 from left to right and the last row should have 13 , 14, 15 from left to right , with the blank space ending up back in the lower right corner.

-

The number of starting positions

The standard version has 15 pieces and a gap, giving a maximum of 16! positions. This limit is not reached because the permutation must be even or odd depending on the position of the gap. There are therefore 16! / 2 = 10,461,394,944,000 positions.

Wikipedia

Solvability

in 1879 Johnson and Story used a parity (even or odd) argument to show that half of the starting positions for the n-puzzle are impossible to resolve, no matter how many moves are made. This is done by considering a function of the tile configuration that is invariant under any valid move, and then using this to partition the space of all possible labeled states into two equivalence classes of reachable and unreachable states.

The invariant is the parity of the permutation of all 16 squares plus the parity of the taxicab distance (number of rows plus number of columns) of the empty square from the lower right corner. This is an invariant because each move changes both the parity of the permutation and the parity of the taxicab distance. In particular if the empty square is in the lower right corner then the puzzle is solvable if and only if the permutation of the remaining pieces is even.

Johnson and Story also showed that the converse holds on boards of size m×n provided m and n are both at least 2: all even permutations are solvable. This is straightforward but a little messy to prove by induction on m and n starting with m=n=2. Archer (1999) gave another proof, based on defining equivalence classes via a hamiltonian path.

Wilson (1974) studied the analogue of the 15 puzzle on arbitrary finite connected and non-separable graphs. (A graph is called separable if removing a vertex increases the number of components.) He showed that, except for polygons, and one exceptional graph on 7 vertices, it is possible to obtain all permutations unless the graph is bipartite, in which case exactly the even permutations can be obtained. The exceptional graph is a regular hexagon with one diagonal and a vertex at the center added; only 1/6 of its permutations can be obtained.
For larger versions of the n-puzzle, finding a solution is easy, but the problem of finding the shortest solution is NP-hard. For the 15-puzzle, lengths of optimal solutions range from 0 to 80 single-tile moves or 43 multi-tile moves. The multi-tile metric counts subsequent moves of the empty tile in the same direction as one.

The symmetries of the fifteen puzzle form a groupoid (not a group, as not all moves can be composed); this groupoid acts on configurations.

Alternate proof

In an alternate view of the problem, we can consider the invariant to be the sum of the parity of the number of inversions in the current order of the 15 numbered pieces and the parity of the difference in the row number of the empty square from the row number of the last row. (Let's call it row distance from the last row.) This is an invariant because each column move, when we move a piece within the

same column, changes both the parity of the number of inversions (by changing the number of inversions by ±1, ±3) and the parity of the row distance from the last row (changing row distance by ±1); and each row move, when we move a piece within the same row, does not change any of the two parities. Now if we look at the solved state of the puzzle, this sum is even. Hence it is easy to prove by induction that any state of the puzzle for which the above sum is odd cannot be solvable. In particular, if the empty square is in the lower right corner (even anywhere in the last row) then the puzzle is solvable if and only if the number of inversions of the numbered pieces is even.