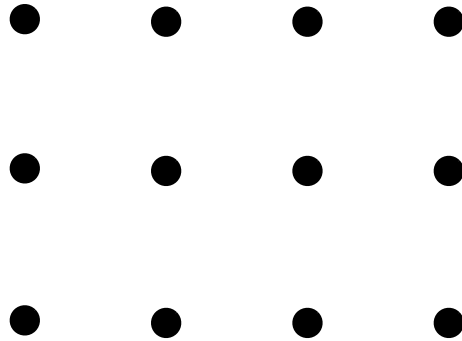


## The 12 Dot Puzzle

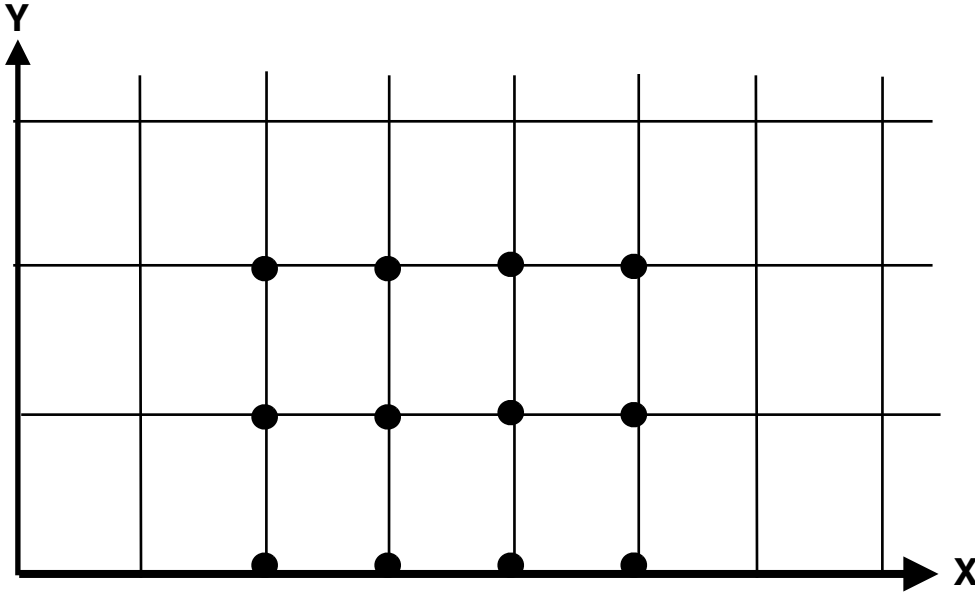
Use a pencil to draw five continuous straight line segments which go through the middle of all 12 dots without taking the pencil off the paper. You must also start and end at the same place.



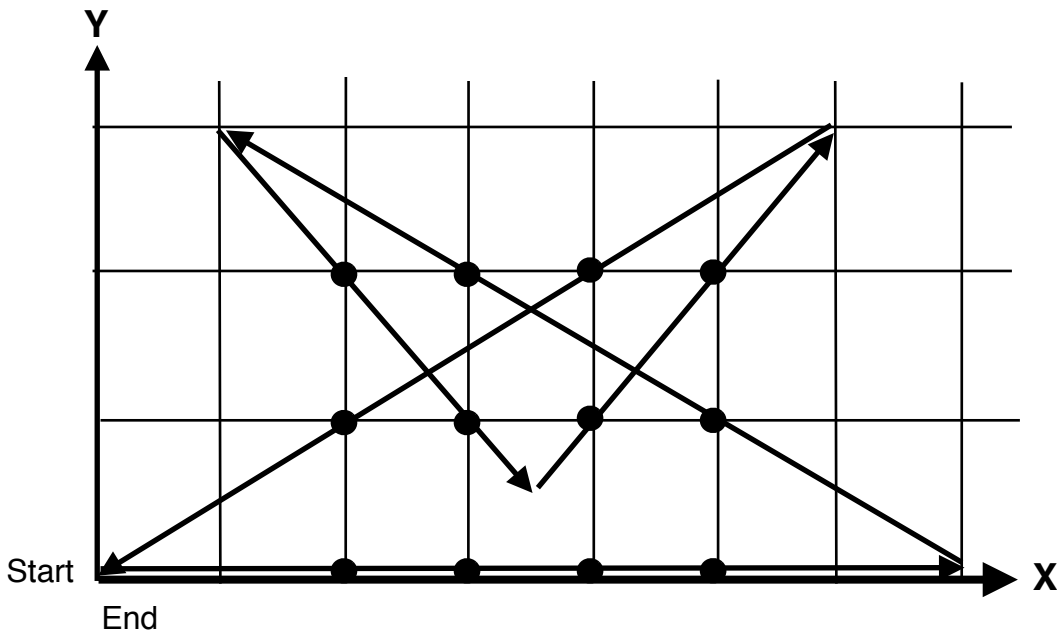
## Coordinate Grid Solution

Most students will have a knowledge of the x y axis and the use of ordered pairs. The solution listed below makes use of this knowledge.

Place the 3 by 4 grid of dots on a coordinate grid as shown below.



- 1: Starting at  $(0,0)$  and draw a line segment to  $(7,0)$ .
- 2: From  $(7,0)$  draw a line segment to  $(1,3)$
- 3: From  $(1,3)$  draw a line segment to  $(3\frac{1}{2}, \frac{1}{2})$
- 4: From  $(3\frac{1}{2}, \frac{1}{2})$  draw a line segment to  $(6, 3)$
- 5: From  $(6,3)$  draw a line segment to  $(0,0)$



One difficulty people have in solving the puzzle is the tendency to make the incorrect assumption that the line segments must stay within the perimeter of the 9 dots. Many people also make the incorrect assumption that each line segment must start and end on a dot. These initial false assumptions cause the person to limit themselves to a point the solution is not possible. No matter how many times they try to draw four straight line segments without lifting the pencil there is always a dot remaining that was not crossed. You will often see the person trying the same pathway many times. Once they drop that limiting thought of a boundary (or are told they can do so) the solution seems to be found very fast.

A few free thinkers may try to use curved "line segments" to get a solution. That leads to a good discussion about setting initial definitions. Some would say the definition of a line segment means that it is straight. Others may argue that if you mean straight you should say so.

Before you start to solve a problem you should examine carefully any assumptions that you may be imposing on yourself that are not actually stated in the problem. The best way to do that is to use clarifying questions.

Can the line segments cross?

Do I need to start at a dot?

Can a dot be crossed more than once.

Can a dot be crossed or touched more than once

Must the line segments be straight?

Is there a limit as to how long each line segment can be?

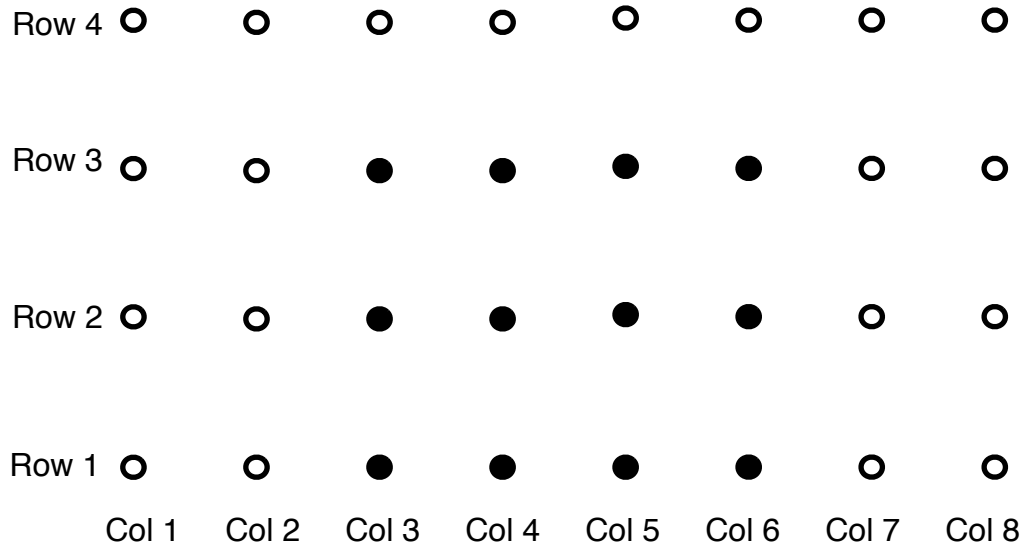
The reason for the coordinate grid being used as part of the solution is based on the need for an easy way to describe the location of the starting point and the length of the segments. If the distance between dots is 1 unit then you need to start 2 units to the right of the dot at the bottom left side of the puzzle and continue 2 units past the dot on the bottom left side of the puzzle. The second line needs to start at this point and end at a point 1 unit to the left and 1 unit above of the upper left most dot on the puzzle. The third line needs to start at this point and end at a point that is midway between the 4 dots in the middle of the puzzle on the bottom 2 rows. It seems clear that this type of description is difficult to understand. This is a classic example of the power of a coordinate system. No lengths or angles or descriptions of locations that are not on a dot in the puzzle are required. It makes the pathway easy to describe.

In fact, the solution is the reason for giving this puzzle to students. Many students will not find the solution without a series of hints. First suggest that put the dots on a grid. Since they may need to go outside the dots on the puzzle to solve the puzzle suggest they leave a couple of rows on either side. From there you can suggest starting points and other pathways by using ordered pairs. This is a good practice in the use of ordered pairs.

## Solution using rows and columns terminology

If students are not familiar with ordered pairs and the  $x$  and  $y$  coordinate system the following solution is provided.

Add open dots to create the 4 by 12 grid shown below. Label the rows and columns as shown if desired.



- 1: Starting at the dot at Row 1 Column 1, draw a line to the dot at Row 1 Column 8
- 2: From there draw a line to the dot at Row 4 Column 2
- 3: From there draw a line to a point this is in the middle of the square of 4 dots at mid-point between the dots at Row 1 Column 4, Row 1 Column 5 and Row 2 Column 4 and Row 2 Column 5.
- 4: From this midpoint, draw a line to the dot at Row 4 Column 7
- 5: From there, draw a line to the dot at Row 1 Column 1

