

# 1089 Number Magic

Write the number 1089 on a piece of paper, fold it, and place it on the desk.

Give a student a piece of paper (or a small pad) and ask the student to write down a three-digit number on the paper. Say that to make sure their the number is a large one they need to make the first and last digits (ones and hundreds places) at least 2 digits apart “like 2 and 5 or 1 and 7” Emphasize that all the digits must be different and that the first and last digits must differ by at least two.

After the student has written down the three-digit number, ask him to write a new number by switching the ones and the hundreds digits of the first number. Ask the student to select the largest of the two numbers and SUBTRACT the smallest of the two numbers from it to get a new number.

## Example:

They pick any 3 digit number where all the digits are different and that the first and last digits differ by at least two.

They pick 729

They write a new number by switching the ones and the hundreds digits of the first number.

They write 927

They SUBTRACT the smallest of the two numbers from the largest to get a new number.

$$\begin{array}{r} 927 \\ - 729 \\ \hline 198 \end{array}$$

Once this is done, ask the student to reverse their new number, switch the ones and the hundreds digits and write their new number under the one they just found b y subtracting. Then ask the student to draw a line under these two numbers and ADD the two numbers.

$$\begin{array}{r} 198 \\ + 891 \\ \hline 1089 \end{array}$$

Have them open the prediction paper you put on the desk at the start of the trick. Your prediction of 1089 will always be correct.

**Extension 1:** You can make this more magical by picking a book and placing it on the desk at the start of the trick. In advance I have looked up the ninth word on page 108. I ask the student to turn to the page that is represented by the left most 3 digits of their final number. The thousands, hundreds and tens digits. Say "If your number is 2, 163 then turn to page 216, Now have them look at the ones digit of their number. Ask them to start at the top of the page on the left and count each word until they get to the word that is the same number of words as their ones digit. Say "If the ones digit of your number is a 3 then start at the upper left side of the page and count to the third word and remember that word. Read their mind and state that word.

**Extension 2:** Place 4 books on the desk and have the student select 2 of them. If they select the pair with the correct book use that pair. If they select the other pair say "we will eliminate these two and use the remaining two books. Do the same thing with the remaining two books to get to the student to select the book you want. This is called The Magicians Force. Then have them find their word using the selected book.

**Extension 3:** Place 3 books on the desk and have the student select any 1 of them. Have the word for each book written down before hand and state the correct word.

**Extension 3:** Have 3 students each do the trick at once without observing each others numbers. Place 3 books on the desk and have each student select any 1 of them. Have the word for each book written down before hand and state the correct word for each student. **DO NOT ASK THEM TO STATE THEIR NUMBERS.** Even though they all have the same number they all get different words so it seems more magical. This is a good effect and the one I use when I present the trick.

**NOTE:** I always have the students perform number tricks with a second student **watching** the first. It gets more students involved and allows one student to check each others work. This helps ensure that their final number is correct.

**NOTE:** There are some limits to the choice of the first 3 digit number. How you start the trick determines how you decide to work around this issue. The procedures I use are designed to make the presentation very simple and works around the limitations almost as if they did not exist. Using one of the extensions makes it a complete trick

The possible limitation on the first number chosen may be important to a math teacher so I list them next. Do not skip this trick due to the limitations. The trick is easy to do, requires no real preparation and no material to print out or hand out. Just have the student get out a paper and pencil and you start the trick. It does not get any easier than that.

## Mathematical insights

This trick is presented in almost every basic book on number magic. It cannot be repeated as a magic trick because the answer is always 1089. but with extensions 3 you can do it for more than one student as long as you do not have them tell the class what number they arrived at.

It is important to note the mathematical issues with the selection of the 3 digit number. Many books do not mention these limitations.

**Issue 1.** You may not want the 3 digit number you select at the beginning of the trick to have a zero as the ones digit. The student may be confused when they switch the ones and hundreds digits and subtracts the two numbers. The trick works if the ones digit is a zero so I do not eliminate that as a possible choice. If you start with 730, you reverse it to 037 you need to ignore the leading zero giving 37, and subtract 37 from 730 to give 693. Then  $693 + 396 = 1089$ , as before.

**Issue 2.** When the student switches the ones and the hundreds digits one number will be larger than the other. For the subtraction to make sense for students the larger number must be listed above the smaller number when they subtract the two numbers. There are two ways of working around this.

A. Just have the student pick any 3 digit number and write it down. Then have them create the second number by switching the ones and hundreds digit and write that number down. Now ask the student to select the largest of the two numbers and SUBTRACT the smallest of the two numbers from it to get a new number. At the most you may need to suggest that they write the larger one and then the smaller one below it. That is the method I use.

B. Require that the first number be written with the different digits in decreasing order. That is what many books suggest. This forces the second number to be less than the first when you switch the ones and hundreds digits.

**Issue 3.** The original number selected cannot have the same ones and hundreds digits. The subtraction would result in zero. The requirement that you select a three digit using "3 different digits" get around this issue.

**Issue 4.** The trick does not work as well for 201, 312, 423, 534, 645, 756, 867 and 978. The trick works in this case but if the ones and the hundreds digits are 1 apart the difference between the numbers will be 99. You would need to add a 0 to the hundreds place before you reverse the number and add the two numbers to get 1089. If you want to eliminate this restriction just be sure to warn the student to add a zero to the hundreds place if needed. I get around this by saying that to make sure their number is a large one they need to make the first and last digits (ones and hundreds places) at least 2 digits apart.

## Here is the trick in a condensed wording.

Pick a 3 digit number with every number a different digit. Make sure the number is a large one by making the first and last digits (ones and hundreds places) at least 2 number apart. Create a new 3 digit number by switching the ones and hundreds digit. Subtract the smallest number from the largest. Take this new number and create a 4th number by switching the ones and hundreds digits. Add these last 2 numbers. The total is always 1089.

## Here is an algebraic proof of the trick:

Let one of the 3 digit numbers be  $A B C$  and the second will be  $C A B$

We did not allow  $A = C$  in the wording of the problem.

We will select  $A B C$  to be the largest of the 2 numbers so  $A > C$

$A B C$  can be represented in base 10 values as  $100A + 10B + C$

When you switch the ones and hundreds digits you get  $C A B$

$C A B$  can be represented in base 10 values as  $100C + 10B + A$

When you subtract  $A B C - C B A$  using base 10 notation you get

$$A B C - C B A = (100A + 10B + C) - (100C + 10B + A) = 99A - 99C = 99(A - C).$$

Since the first and last digits in  $A B C$  differ by at least 2, then  $(A - C)$  is either 2, 3, 4, 5, 6, 7, 8, or 9.

So,  $99(A - C)$  is one of the following:

$$99 \cdot 2 = 198 \quad 99 \cdot 3 = 297 \quad 99 \cdot 4 = 396 \quad 99 \cdot 5 = 495 \quad 99 \cdot 6 = 594 \quad 99 \cdot 7 = 693 \quad 99 \cdot 8 = 792 \quad 99 \cdot 9 = 891$$

That says  $99(A - C)$  is one of the following 198, 891, 297, 927, 396, 936, 495 or 594

We want to add this number to its reverse. Repeat what we did before and apply it to this number.

Let's call the number  $X Y Z$  which is  $100X + 10Y + Z$  and  $X Y Z$  is  $100Z + 10Y + X$

We want to add  $X Y Z$  to  $Z Y X$

Looking at the possible numbers for  $X Y Z$  above, we notice that the middle number  $Y$  is always 9.

We also notice that the base ten values of the first and third digits always add up to 9, i.e.,  $X + Z = 9$ .

So,  $X Y Z + Z Y X$

$$= 100X + 10Y + Z + 100Z + 10Y + X$$

$$= 101X + 101Z + 20Y$$

$$= 101(X + Z) + 20Y$$

$$= 101(9) + 20(9)$$

$$= 909 + 180 = 1089$$

Remember the trick is worded so that the ones and hundreds digits are at least 2 digits apart. The trick does not work as well for some students of the number chosen is 201, 312, 423, 534, 645, 756, 867 or 978.

### **Why to you limit the different between the hundreds and i-ones digits to 2 or more?**

When the difference between the hundred and ones digits is 1 you get a 99 for the difference in the two 3 digit numbers. You have to have the student put a zero in the hundreds location to make it a three digit number before reversing the number and then adding 99 and 990 to get 1089. For example, if you start with 392, reverse to 293, and subtract, you get 99. Make this 099 before you reverse it to get 990, and add 99, getting 1089 as predicted. This will happen when the difference between the hundred and ones digits is 1. For this reason 201, 312, 423, 534, 645, 756, 867, 978 don't work as well as when the difference is 2 or more.

### **1089 Facts.**

1089 is 33 squared. it is the first reverse divisible number where a smaller number divides evenly into it's reversed digits. The next is 2178. There are only four 4 digit numbers that evenly divide into their reverse.

Multiplying the number 1089 by the integers 1 to 10 produces interesting patterns.

any 2 Multiples that add to 10 give 4 digit that are reversal of each other.

Example  $1 * 1089 = 1089$  and  $9 * 1089 = 9801$

$$\begin{aligned}1 * 1089 &= 1089 \\2 * 1089 &= 2178 \\3 * 1089 &= 3267 \\4 * 1089 &= 4356 \\5 * 1089 &= 5445 \\6 * 1089 &= 6534 \\7 * 1089 &= 7623 \\8 * 1089 &= 8712 \\9 * 1089 &= 9801\end{aligned}$$

Note the pattern in the answer columns. They are consecutive chains of the digits 0 to 9.

David Acheson from Oxford University is the author of a book about 1089 and other mathematical patterns titled 1089 And All That: A Journey into Mathematics

### Extra for the Super Expert:

This same procedure could be applied to 4 or 5 digit numbers with some extra work. In the case of 4 or 5 digit numbers there are 4 possible outcomes but each of the final answers is easily determined from the initially chosen number. For a standup trick, the effect is amplified by asking the student to call out the digits of the number as they write it on their paper. When they say the last digit you can immediately write down the final result of the procedure. When the student finishes doing the calculations, their number matches yours.

The palindromic numbers, i.e., the numbers that read the same forward and backward, must be excluded from the demonstration as they always and immediately lead to a 0.

Here is a summary of the cases for the 4 digit number A B C D

- 1 10989: ( $a > d$ ,  $b = c$ ).
- 2 10890: ( $a > d$ ,  $b > c$ ).
- 3 9999: ( $a > d$ ,  $b < c$ ).
- 4 990: ( $a = d$ ).

Note that the condition " $a > d$ " is not actually restrictive as it is always possible to mentally swap the number and its reverse.

Here is a summary of the cases for the 5 digit number A B C D E

- 1 109989: ( $a > e$ ,  $b = d$ ).
- 2 109890: ( $a > e$ ,  $b > d$ ).
- 3 99099: ( $a > e$ ,  $b < d$ ).
- 4 10890: ( $a = e$ ).
- 5

For example, the procedure applied to 36785 will give 109890.

To check this, observe that this number needs to be mentally switched with its reverse. After that switching  $a = 5$ ,  $b = 8$ ,  $c = 7$ ,  $d = 6$ ,  $e = 3$ . Since  $b(8) > d(6)$ , we have the second case, with the final answer 109890.

#### Example

$$\begin{array}{r} 58763 \quad a = 5, b = 8, c = 7, d = 6, e = 3. \quad A > E \text{ and } B > D \text{ so the answer is } \mathbf{109890} \\ - 36785 \\ \hline 21978 \\ + 87912 \\ \hline \mathbf{109890} \end{array}$$

## Computer code for the trick using Matlab

First, you need to know the difference between numbers and strings. Sometimes they look similar, but there's an important difference. You can do arithmetical operations with numbers, but you cannot do them with strings. Function `num2str` changes numbers into strings, so `num2str(531)` results in string '531'. Note that if you type `size(531)` on the command window, Matlab will answer 1 1, meaning that you have a 'matrix' with one row and one column only (that happens to be a scalar!). If you type `size('531')` - using single quotes for the parameter - on the command window, Matlab will answer 1 3, meaning that you have a matrix with one row and three columns. It's a string with three elements or characters, even though you see it as a number.

Function `str2num` changes strings into numbers. A `str2num('123')` will produce the scalar number 123.

Function `fliplr` flips strings (not numbers) from left to right, or invert them so to speak.

`fliplr('351')` will produce the string '153'.

**This is the code that could perform the trick.**

```
clear, clc
% this is your initial number
a = 123
% change it to string
as = num2str(a)
% flip the string
asi = fliplr(as)
% change back the inverted string to number
b = str2num(asi)
% get the difference
c = abs(a-b)
% change to string
cs = num2str(c)
% flip the string
csi = fliplr(cs)
% change back the inverted string to number
d = str2num(csi)
% add and get 1089
e = c + d
```

Note that there are **two instructions that were not considered in the code**. First, the digits should be different, and second, if the resulting difference is two-digit only, you should write a 0 at the beginning. Those issues were not considered, but now you could design an iteration to see which 3-digit numbers didn't produce 1089 after the code given.

The improved code is:

```
correct = 0;
wrong = 0;
% These are your starting numbers
for a = 100 : 999
    % flip them
    b = str2num(fliplr(num2str(a)));
    % get the difference
    c = abs(a-b);
    % flip the difference
    d = str2num(fliplr(num2str(c)));
    % add and get 1089
    if (c + d) == 1089
        correct = correct + 1;
    else
        %if not, display count, starting number and result
        wrong = wrong + 1;
        disp ([wrong a c+d])
    end
end
```

```
correct
wrong
```

And you'll get the list of numbers that didn't have different digits or that produced a two-digit difference... Besides, you counted the "correct" numbers and displayed and counted the "wrong" ones.

```
1   100   198
2   101     0
3   102   198
4   110   198
5   111     0
...
...
258  989     0
259  998   198
260  999     0
```

```
correct = 640
wrong = 260
```



This is the code for the 1089 trick.

It includes error handling as well as output to the screen.

```
<?php

// Declare a unique 3 digit number
$num = 987;

// Remove spaces to account for user typing error
$numtrim = str_replace(" ", "", $num);

// error check for numbers that do not have 3 digits
if (strlen($numtrim) != 3){

echo "Your number must be 3 digits long, please enter a different
number<br />";
return;
}

// Split each digit of the number entered into an array.
$digits = str_split($numtrim);

// error check for 3 digit numbers that do not have unique digits.
Digits[0] is the first digit, Digits[1] is the second digit, and
Digits[2] is the third digit
if ($digits[0]==$digits[1] || $digits[0]==$digits[2] || $digits[1]==
$digits[2]){

echo "Your number must contain 3 <i>unique</i> digits, please enter
a different number<br />";
return;
}

// Declare a shortcut function to underline text
function uh($text){
echo "<u>" . $text . "</u><br />";
}

// Declare another number 2 which reverses the digits of number 1
$numtrim2 = strrev($numtrim);

// Print out text
```

```

echo "Show the 1089 trick using the number " . $numtrim . " and " .
$numtrim2 . "<br />";
echo $numtrim2 . " is used as the reverse digit order of " .
$numtrim . "<br /><br />";

// Determine the max and min of the number entered and the reverse
digits number
$numenum1 = max($numtrim,$numtrim2);
$numenum2 = min($numtrim,$numtrim2);

// Subtract the smaller number from the largest number
uh("Step 1: Subtract our smallest number " . $usenum2 . " from our
largest number " . $usenum1);
$subnum = $usenum1-$usenum2;

// The strrev function reverses a string, so 123 for example would
become 321
$subnum2 = strrev($subnum);
echo $usenum1 . " - " . $usenum2 . " = " . $subnum . "<br /><br />";

uh("Step 2: Reverse this number:");
echo "The reverse of " . $subnum . " is " . $subnum2 . "<br /><br /
>";

// Now the reversal of digits is added to the original number
difference
uh("Step 3: Add the result in Step 2 to the result in Step 1:");
$answer = $subnum+$subnum2;
echo $subnum . " + " . $subnum2 . " = <b>" . $answer . "</b><br /
><br />";

// Print out final confirmation message
echo "As promised, this equals 1,089!!!!<br /><br />";
?>

```